6

John Adams

THE YOUNG ALGEBRAIST'S COMPANION,

O R,

A NEW and EASY GUIDE to

ALGEBRA;

Introduced by the Doctrine of

VULGAR FRACTIONS:

Designed for the Use of SCHOOLS, and such who, by their own Application only, would become acquainted with the Rudiments of this noble Science:

Illustrated with

Variety of numerical and literal Examples, and attempted in natural and familiar Dialogues, in order to render the Work more easy and diverting to those that are quite unacquainted with *Fractions* and the *Analytic* Art.

The SECOND EDITION Corrected.

To which is added,

An APPENDIX

On the Rudiments of Quadratic Equations, and a new and easy geometrical Definition of the Difference between the solid Content of the Cylinder, and the Parallelopiped proved by the Pen.

By DANIEL FENNING, *of the* ROYAL-EXCHANGE ASSURANCE.

LONDON:

Printed for G. KEITH at *Mercers Chapel, Cheapside*, and J. ROBINSON at *Dock-Head, Southwark*. 1751.

DEDICATION.

To the Honourable the GOVERNORS *and* DIRECTORS *of the* ROYAL EXCHANGE ASSURANCE COMPANY.

GENTLEMEN,

IT being not only a common, but a very juſt Obſervation, that the Generality of *Dedications* are carried to too high an Extreme of flattering Encomiums, I perſuade myſelf you will the more readily accept of one in a *plain* Dreſs only, as I am ſenſible it is no-ways agreeable to your *Honours* to be flattered.

When ſolicited by ſeveral Gentlemen in Town and Country to publiſh the following Tract, I was not long conſidering at whoſe Feet to lay it: The Preſumption indeed was ſo great, that though the Thoughts of former Favours in ſome Meaſure encouraged me, yet it was with Reluctance I preſumed to beg *this*; and your permitting me to ſend it Abroad under ſuch Proteſtion, makes it difficult to ſay which is greater, the Honour, or the Kindneſs you have done me, ſince from the *One* I ſhall naturally reap the Benefit of the *Other*.

Even Books of *common Arithmetic* only have ſeldom wanted Patronizers, but Gentlemen of the firſt Claſs have more eſpecially condeſcended to protect and maintain the more ſuperior Parts of *Mathematical* Learning, not only from the Pleaſure and Satisfaction that ariſes even from the very *ſpeculative* Knowledge of them, but by being ſo uſeful in exerciſing the Faculties, and bringing the Mind into a juſt and proper Method of Reaſoning, were always thought worthy the Study of the beſt of Men, are eſteemed as pretty Decorations

tions and Accompanyments to other Branches of Learning, and for Want of which, Education is always counted Something the leſs complete.

It is ſuperfluous indeed to mention this; ſince it is evident your *Honours* were ſenſible of the Truth of it, from your Readineſs to promote a Work wrote by one who at beſt profeſſes himſelf but a *Novice* in the thorough Knowledge of this extenſive Study: So that your Love for Arts and Sciences is the more conſpicuouſly ſeen, and you will appear upon the Liſt with thoſe worthy Patriots, whoſe Generoſity and Benevolence prompt them daily to ſupport and encourage all Sorts of Literature.

As I am ſenſible your *Honours* expect Nothing from me but Thanks and Duty, the firſt I moſt humbly return; and as I am conſcious I have hitherto been faithful and diligent in your Service, I hope I ſhall always endeavour, to the beſt of my Abilities, to continue ſo, in due Senſe of the many Kindneſſes I have received from you.

That the ſame kind Providence that recommended me to your *Honours* Favour, may continue to each Member the Bleſſing of Health, and *that* of Proſperity to the Company in general, is, without Doubt, the hearty Deſire of many; but of none more than,

Gentlemen,

Your Honours moſt oblig'd,

and obedient,

humble Servant,

D. FENNING.

PREFACE.

Kind READER,

MANY *and great are the Advantages that arise from the Knowledge of common* Arithmetic *only, to be Master of which requires a great deal of Time and Practice; but* Algebra *so far exceeds it, that were it not but that some sensible Progress must be made in the* Former, *it would hardly bear any Comparison with the* Latter.

It may indeed be thought a needless and no less vain Attempt to offer any Thing of this Sort to the Public, *that has already been treated of by so many eminent Authors, as if more could be said upon the Matter than has already been done. But it is to be observed that the following Work is designed only as a Preparative to the right understanding of other Authors; so that perhaps, upon due Consideration, the Undertaking may be found to be neither impertinent nor improper: For the Question is not,* Are there not Books enow extant? *but,* Whether the Generality of them are fit for Learners, so that they may be capable in a short Time of themselves to solve Questions with the Help of a Master? *This I think is the grand Point to be considered.*

I do

I do not mean in the least to lessen those Authors, for whose Works I would be thought to have the highest Regard; but from the little Progress that most make who undertake to learn Algebra *barely by Book, one would conclude (and that without the least Detraction) that the Rules, Instructions, and Examples, are not so plainly suited to the Capacities of Learners as could be wish'd.*

A great many are very ambitious to purchase for their very first Book some great and noted Author (and indeed such must be consulted, if we would become Masters of any Art or Science;) but it is a common Mistake to think they are best. and most to be depended upon in the Rudiments or fundamental Principles of such Arts or Sciences. It is evident there is Something more in this than barely asserting it; for if it were so, what should be the Reason that so many inferior Authors (perhaps of ten Times less Knowledge in the Art or Science itself) have wrote, and do still continue to write Volumes, for the better understanding and explaining their Works?

If indeed there was no Occasion for this, *then have they spent their Time in vain, and their Labours to no Purpose: But certainly they have not done so; it is what was wanting, and therefore a very necessary Undertaking; because by thus doing the Unlearned have and still may, become acquainted with such* Mathematical Books, *which otherwise they never would understand, without having a Tutor always at their Elbow; and then indeed* Pardie's Geometry *may be understood as well as* Leadbetter's Mathematical Companion, *or* Boad's Artium Principia: *And* Wolfius *and* De Billy *as well as* Cocker *or* Hammond, *but not without* this *or some such like Advantages. Besides,*

It has long since been allow'd, that it is none of the easiest Things for Men of profound Learning to write within the ordinary Compass of common Capacities: Their Knowledge will very rarely suffer them to stoop to the

the Underſtanding or Conception of ſuch: So that what Dryden *ſays of* Love *and* Wiſdom *together, may in another Senſe be applied to the Caſe before us:*

"The Proverb holds that to be wiſe and love,
"Is hardly granted to the God's above."

For indeed, to Men of ſuch extraordinary Parts and Abilities, every Thing of this Sort is ſo very eaſy, that whatever they propoſe, be it either Precept or Example, they can diſtinguiſh it ſo many different Ways, that from hence they conclude it is very eaſy to be underſtood by others; on which Account they are in general not only ſo ſublime, but withal ſo conciſe, that it is well known, that not only moſt of their eaſieſt Problems *(as they are pleaſed to call them) but even their Demonſtrations, require a further Demonſtration and more eaſy Explanation, before moſt Learners can of themſelves form a clear and diſtinct Idea of the Nature of the Propoſition; at leaſt it will coſt them many a weary Hour, and ſometimes Days and Months, to underſtand the whole Operation, though they apply ever ſo cloſely and aſſiduouſly; and all this for Want of a few Words applied in a free, natural, and eaſy Manner. Indeed, when we conſider theſe Things, we cannot but ſay it is Pity a diligent Reader ſhould ſpend his Time to ſo little Purpoſe.*

As Algebra *is noted for its Excellency, ſo it is for its Difficulty, and therefore ſeveral eminent Authors have been pleaſed to call it a* dark *and* dry *Study; the Meaning I apprehend to be this, becauſe the Learner goes on a long Time through a Series of Rules and Examples, which, though he be ever ſo perfect in, yet ſees no Reaſon for what he has done, nor receives any Reliſh or Satisfaction till he comes to put them in Practice in the ſolving of a few Queſtions; and even* then, *if the Author does not give a Reaſon for almoſt every Step of the Operation,*

Operation, the young Tyro *will not ſo ſoon underſtand the Work as he may imagine, but is often at a Loſs, and ſometimes totally ſtopped.*

Now if the young Algebraiſt *may poſſibly by ſome Authors proceed thus far, and for Want of better Aſſiſtance can go no further, what can be thought of ſuch Books as have not ſufficient Inſtructions for the very Rudiments only: And that there are ſuch is evidently known to too many, as will appear from the Words of an Author himſelf*. — "I have always been of Opi-" nion, ſays he, that* Algebra *ſhould not be entangled " with a great Number of Precepts; the Science is " dark enough, without adding to it new Obſcurity " by the Confuſion of different Operations,* &c." — *Had he ſaid* confuſed *Operations indeed it had been to the Purpoſe; but what he means by calling different Operations a Confuſion, I know not: For Reaſon (I think) tells us, that a Variety of Examples plainly demonſtrated, is the moſt ready Way to remove Obſcurity, as it is the only Means to prevent a Learner's being entangled; becauſe without Examples he would have but imperfect Ideas, and conſequently could never underſtand the* practical *Part of any Art or Science: It is therefore not only diſingenuous to own a Thing to be hard, and not withal to give ſufficient Rules and Examples to render it eaſy when in our Power; but it is as abſurd to think it ſhould ever be rightly underſtood without theſe.*

He is no great Arithmetician *who will not allow the Doctrine of* Fractions, *and the* Extraction *of* Roots *to be ſomething more difficult than the lower Rules of common* Arithmetic; *for which Reaſon the Directions are ſo many the more, and ſo much the fuller: If then to*

* *It is of no Signification to mention the Author's Name. Thoſe that have him by them may ſee the ſame Words; and I make no Doubt but they will find he has kept up to them throughout his Work.*

to underſtand theſe only, ſuch a Number of Caſes and Examples are neceſſarily required, is it reaſonable to ſuppoſe Algebra *is to be learnt without, or by a few only? No ſurely; for look but into the Works of the* unparallelled Saunderſon, *you will there find Precept upon Precept, and Example upon Example; and were it ſo that two Volumes in* Quarto *could be purchaſed by every Lover of this Science, they would (in all Probability) make more* Algebraiſts *than* all *the Books extant would for ſome Years put a Stop to any further Pretences, and may be ſaid to be almoſt a* finiſhing *Stroke upon the Subject.—But this cannot be done.*

Now, contrary to this, ſeveral Authors are ſo over and above ſhort and intricate, that it is almoſt impoſſible for a Beginner to learn ſo much as the Algorithm, *much leſs the* Algoriſm *.

What may be done by one in a Thouſand is not eaſily accounted for. Heaven has favoured ſome with a ſurprizing quick Apprehenſion, a penetrating Judgment, and tenacious Memory; and if to theſe we add the Advantages *of Time, together with the* Delight *and* Operoſity *that ſuch a one may poſſibly take in any Branch of Learning, ſuch a Man as* this, *I ſay, cannot fail to make ſenſible Improvements from the moſt ſublime Authors, and the moſt intricate Demonſtrations: But can we expect to find this at every Door? Very few have two of all theſe Advantages, and therefore* that *which is but* A, B, C *to the* one, *is as* Greek *to the* other.

From a due Conſideration of theſe Things, it is eaſy to perceive that a Book wrote in a plain and familiar Manner, and with a moderate Price, has been long wiſhed for and expected; the Want of which (as ſeveral

* Algorithm *ſignifies the firſt Principle, and* Algoriſm *the practical Part, or knowing how to put the* Algorithm *in Practice.*

veral Schoolmasters, and others my Acquaintance, have often said) has been a very great Discouragement to Learners in their intended Design and Pursuit of Algebra.

The most rational Method to make any one Master of any Art *or* Science, *is certainly to introduce it to him in a natural Order, and to teach him at* first *only so much of it as is most necessary and consistent with the Foundation*; *and when he understands the Fundamentals, he will soon be able to conquer the more difficult Parts, and may pursue the Study in a more learned Manner, if his Fancy, Inclination, or Profession, shall incline him so to do.*

Such then is the Design of the following Sheets, to give the Learner a true Notion of Simple Equations *only*; *and to make it as useful to him as I possibly could, I have added that Part which treats of* Vulgar Fractions, *lest he should not be acquainted with so necessary a Step: And though I have laid down every Thing in the first four* Algebraic *Rules as plain as I am able, still, that the young Practitioner may not be at a Loss in working the* Problems, *I have there recalled him back to his former Work, have given him the Reason* gradatim *(Step by Step) throughout the whole Operation, that he way the more readily understand the Nature of what he has been doing.*

I expect some critical Adepts *may say, there was no Occasion to be so very particular*; *but let them be told once more, it is designed only for Learners, (though they themselves, perhaps, may find some Things in it not altogether unworthy their Notice*;) *and I persuade myself, upon this Consideration, I have not dwelt too long upon any one Thing that requires a clear Demonstration, if it were only for this Reason, that considering the many Rules and Examples the Learner has got to go through before he can put them in Practice,*

and the Difficulty of putting them in Practice to his own Satisfaction, an Author may be, and often is, too ſhort *for moſt, but he never can be too* plain *for any; and I think the* practical *Part of* Algebra *has already been ſufficiently prov'd to be inconſiſtent with too much Sublimity and Conciſeneſs. Again,*

Does not Reaſon itſelf tell us, that Arts *and* Sciences *are not like* Hiſtory. *A few Words in the different Circumſtances and Parts of the* Narration *give us a general Idea of the Whole: But* here *it is quite otherwiſe; the Reader muſt have Words as it were continually multiplied, to underſtand truly what is before him; ſo that* that *which may be called a needleſs* Tautology *in the* one, *is Nothing more than a proper and neceſſary* Repetition *or* Addition *to the* other.

I hope, therefore, thoſe more ſkilled in this Science *will excuſe my being a little prolix, if I tell them I have done it in* Sympathy *to the young Practitioner, ſince I know the dear Purchaſe of ſtudying* Algebra *from conciſe and obſcure Books by woful Experience!*

It is true, I have attempted the Work in Dialogues, which render it more prolix than it would have been without them; but as I had never ſeen any Thing of this Sort wrote in that Manner, I did it partly to avoid the Charge of Plagiariſm; *and it is the general Opinion, that this Way of Writing conveys the Senſe of the Matter ſooner to the* Ideas, *as it unbends the Mind at Intervals, not by turning it aſide from the Subject itſelf, but inſenſibly ſteals upon the Fancy, and renders the Study in a great Meaſure a Diverſion rather than a dry burthenſome Taſk. But a benevolent* Critic *knows,*

"Whoever thinks a faultleſs Piece to ſee,
"Thinks what ne'er was, nor is, or e'er ſhall be."

And a candid Judge, when he conſiders the Scope of the Science, and Deſign of the Author will grant,

That if the Way be juſt, the Conduct true,
Some Praiſe, in Spight of trivial Faults, is due.

Let therefore the ingenuous Reader conſider, that every Day has its Shades, and that my chief End is to ſerve him, and to ſave him Trouble in the Purſuit of this excellent Study, and then he will, for the very Deſign's Sake, forgive thoſe Errors which the Preſs, Want of Time, or Ability, may poſſibly have occaſioned.

Yours, &c.

Royal Exchange,
London, April
28, 1750.

To

To the AUTHOR of the YOUNG ALGEBRAIST'S COMPANION, &c.

SIR,

THERE is Nothing can give a greater Satisfaction to a Lover of the Sciences, than to ſee them handled in a clear and maſterly Manner; and that every Attempt to remove Difficulties, and clear up the Obſcurities of any knotty Part of Science (ſo as to level it to the Capacities of Youth, and at the ſame Time to make it pleaſing as well as inſtructing) muſt be allow'd, by *all*, to be a difficult Taſk. Give me Leave therefore to thank you for the laudable Pains you have taken in your YOUNG ALGEBRAIST'S COMPANION, wherein you have made *that* which was hard and difficult, plain and eaſy to be underſtood, and at the ſame Time, have wrote it in ſo engaging a Manner, that the moſt inattentive Peruſer muſt receive at once both Pleaſure and Information. I am,

SIR,

Your unknown Friend

and humble Servant,

TOWER-ROYAL,
London, Aug. 21,
1750.

SAMUEL HILL,
Philom.

N. B. *The* Author *has had ſeveral ſuch friendly Letters as this, from other Strangers, in the* City *and* Country.

We whose Names are here underwritten, having each of us perused the following Sheets, do allow the Dialogues and Demonstrations to be very natural, and easily adapted; and therefore beg Leave to recommend the Work, as one of the plainest and best suited to the Capacities of young Beginners extant.

Anthony Gilbert,
Peter Dennis, Surveyor,
Thomas Humphreys,
Timothy Langley, Accomptant,
Joseph Simpson, Teacher of the Mathematics,
John Repton, Supervisor,
Abraham De Lire, Teacher of the Mathematics,
John Quant, Writing-Master and Accomptant,
George Coles, Land-Surveyor,
Richard Richardson,
Erasmus Turner,
Zachariah Snaper, Accomptant,
Samuel Hill, Philom.

INTRODUCTION.

CHAP. I.

DIALOGUE I.

Between PHILOMATHES and TYRUNCULUS.

Tyr. —— (*at* Philomathes's *Door*——*knocks.*)
Servant.————

Tyr. PRAY is *Philomathes* within?

Serv. Yes, Sir.

Tyr. Has he Company?

Serv. No, Sir: Pleaſe to walk into the Parlour, my Maſter is quite alone.

Tyr. —— (*within*) I chooſe rather you would let him know I am here.

Serv. —— (*to* Philomathes) Sir, here's a Gentleman deſires to ſpeak with you, if you be at Leiſure.

Phi. Who is it, *Pſapho?*

Serv. I have ſeen the Gentleman before, Sir, but I forget his Name.

Phi. —— (*comes.*)

 Tyr.

Tyr. Dear *Philomathes*, I am your humble Servant.

Phi. *Tyrunculus*, I am yours, and am heartily glad to ſee you. — Come, Sir, pray walk this Way, I beg of you.

Tyr. — Sir ——

Phi. Pleaſe to ſit here, Sir, for you ſeem to be cold.

Tyr. Sir, I thank you; I am a little cold, I confeſs.

Phi. Give me Leave, *Tyrunculus*, to repeat once more, that I am proud to ſee you; and I hope, after the many Promiſes you have made, you are now come on Purpoſe to ſpend an Hour or two with me.

Tyr. Indeed, *Philomathes*, I came with that Deſign, if it be convenient; if not, I beg I may wait upon you at a more ſuitable Opportunity.

Phi. Indeed, Sir, I ſhall conſent to no ſuch Thing; you ſhall not put me off from Time to Time with your Apologies; you ſee I am alone; why then ſhould you think it is inconvenient?

Tyr. Pardon me, *Philomathes*; as I ſee your Books before you, I thought you might be too buſy at this Time.

Phi. Piſh, —— I am never ſo engaged with them, but that I'm always ready to receive my Friend; though I confeſs (when I have not the Pleaſure of an agreeable Companion) they are pretty Company to me of themſelves.

Tyr. That I believe; for come when I will, you are always upon the Study.

Phi. Indeed I am no cloſe Student, *Tyrunculus*; 'tis true, ſome things require more Thought and Conſideration than others, and I believe none more than what I have now been reading.

Tyr. Pray what is that, if I may be ſo free?

Phi. No Secrets at all, *Tyrunculus*; you may look of any of the Books and welcome.

Tyr.

Tyr. So, ſo! *Algebra!* —— *Cocker*, *Hammond*, *Saunderſon*, *Kerſey*, *Ward*, *De Billy*, *Wolfius*, &c. You have a Variety of Authors upon my Word.

Phi. The Science requires it, *Tyrunculus*.

Tyr. I believe ſo, and a fine Science it is; for my Part, I wiſh I underſtood it; but it is ſo hard for a Learner to ſtep into it of himſelf, that I think it is quite tireſome.

Phi. You ſurprize me, *Tyrunculus*; why, you talk'd of learning long ago.

Tyr. I did ſo, and bought two or three good Pieces (as they are call'd) upon it; but I own I am very little the wiſer for all my ſtudying.

Phi. Do not ſay you ſtudied; I fancy you only gave them a careleſs Look or two; and Things of this Nature ſhould be read ſlow, and with the greateſt Attention.

Tyr. I am ſorry, *Philomathes*, you think I have taken no Pains; many an Hour's Reſt have I broke, to learn only the firſt four Rules; and as for Equations, I never could underſtand; and to ſay the Truth, I have laid the Thoughts of it aſide for ſome Months.

Phi. I beg, *Tyrunculus*, you would not be angry; what made me think ſo is, becauſe I know moſt Learners are apt to run over Things too haſtily, and then blame the Author for their not coming at them directly: However, it is plain the Fault is either in you or the Books (if not in both) that you have made no better Improvement.

Tyr. I am apt to think the Fault is in them as much as in me.

Phi. Perhaps ſo; but what ſignifies barely aſſerting it, without giving a Reaſon?

Tyr. Whether I be expert enough to give you a ſufficient one, I will not ſay.

Phi. Let me hear you give ſome Reaſon or other, I beg.

Tyr. Why then, *Philomathes*, two of my Authors treat only of *Algebra* itſelf, beginning with *Problems* directly; the other indeed begins at *Addition*, and proceeds on to the firſt four *Algebraic* Rules, which I learnt pretty well; but when I came to *Algebraic Fractions*, I (not knowing any Thing of *Vulgar*, and he being ſo conciſe) could underſtand very little of them; on which Account, when I came to Equations, I was quite at a Loſs; for I perceive there are very few but what have Fractions, and I know very little of them, except juſt to read them.

Phi. Why indeed, *Tyrunculus*, if you are not acquainted with *Vulgar Fractions*, it is in vain you pretend to ſtudy *Algebra*, for they are the very Baſis and Foundation of it; however, *Algebraic Fractions* are done after the ſame Manner as *Vulgar*.

Tyr. But you will allow them to be much the harder of the two, I imagine?

Phi. Yes, yes, I grant it; for when the Learner is well acquainted with *Vulgar*, he will ſoon underſtand *Algebraic Fractions*; beſides, it will ſave him a great deal of Trouble, for it is impoſſible to reduce an Equation, in order to diſcover the Value of the unknown Quantity, without underſtanding one or both of theſe.

Tyr. Since you grant this then, *Philomathes*, I think you muſt own the Fault to be in the Books rather than in me?

Phi. 'Tis true, the Books you ſpeak of are not fit for Learners; but ſtill, I muſt not indulge you ſo much as to lay all the Fault on them; for, as I ſaid before, though they may not be immediately fit for Beginners, yet they may be very good Books; for you are to conſider, *Tyrunculus*, that ſome Authors

thors ſuppoſe Perſons previouſly to underſtand ſuch and ſuch Rules.

Tyr. You ſay very right, *Philomathes*; but pray did you never hear any beſides me complain of the Difficulty of learning *Algebra* by moſt Authors?

Phi. Yes, a great many, and there may be Reaſons for it; but we ſhould always, in ſuch Caſes, paſs our Sentiments with Reaſon, Caution, and Tenderneſs, and not blame Authors on Account of every ſupine Learner; for it is evident we can have no better, if we go to the Extent of the Science; tho' I muſt confeſs thus far, that, upon the Rudiments, the Generality are a little dark, and too conciſe.

Tyr. Since you own this, *Philomathes*, how then could you ſo ſeverely blame me, that I have made no better Progreſs?

Phi. You are not ſo much to be blamed as I thought for; but ſtill, you are to be blamed in this Reſpect, that you have not conſulted more Books; for he that deſigns to be the Maſter of any Art or Science, ſhould certainly provide himſelf with a Sufficiency for ſuch an Undertaking; for it often happens, that what one Author is deficient in, or treats darkly upon, another ſets forth in a clear and eaſy Manner to be underſtood.

Tyr. A Variety of Books cannot be had, you know, without Expence.

Phi. You make me ſmile to hear you talk of Expence; you know very well you can afford to purchaſe any Books you have a Mind to.

Tyr. That may be, but I ſpeak in Pity to thoſe that cannot; for by Reaſon of this, many Minds lie quite uncultivated, which otherwiſe would make fine Improvements in ſeveral Branches of Learning: This was the Caſe of *Tyro*, when he took in Hand

to learn this Science, and I made him a Preſent of *Cocker* and *Hammond* to encourage him.

Phi. You did very kindly, and could not have made him a prettier Preſent; and he will certainly learn, for he has a very pretty Turn of Thought for Figures.

Tyr. I have heard *Novitius* ſay the ſame, (for they both practiſe together) but yet he ſays, that he never could rightly underſtand in what Manner *Cocker* reduces ſeveral of his Equations; for my Part, I wonder at his Patience.

Phi. Delight, *Tyrunculus*, Delight carries us thro' many Difficulties: But pray, do you remember any particular Queſtion or Equation that he ſeems ſo much puzzled about?

Tyr. That I do not; but I heard him wiſh he had the Happineſs of being acquainted with you, he would have aſk'd you how to reduce a certain Equation or two, which puzzles him pretty much, but that he feared you would take it amiſs.

Phi. Not I, in the leaſt; you know, *Tyrunculus*, I am of no ſuch ſelfiſh Temper; I hate it of all Things; it would be Ingratitude not to communicate that freely, which I received ſo: Baſe and ſordid Spirits will indeed deny their Aſſiſtance, that they may have the Pleaſure of laughing at the Ignorance of thoſe that they ought to have inſtructed.

Tyr. It is very true, *Philomathes*, *Philautus* is of this unhapy Diſpoſition; I heard *Novitius* ſay, a little while ago, he aſk'd him only a ſingle Queſtion, and he would not reſolve it, but gave him very little to anſwer, and ſeem'd, he ſaid, to be affronted; and yet you know they are intimately acquainted, and he always expreſſes the greateſt Regard for *Novitius* in other Reſpects.

Phi. It is very ſurprizing! The various Tempers of Men are not eaſily to be accounted for you know,

Tyr.

Tyrunculus : To be ſure, that Man can be of no Service to any Society, that is not ready and willing to aſſiſt every Member of it, and eſpecially when he is entreated. *Lucretius* was wont to ſay, "That he "would ſeek all Opportunities to communicate "whatever he thought might be ſerviceable to any "Man; and that, if Wiſdom and Knowledge were "given to him with that *Reſerve*, that he might "not impart it to others, he ſaid he would much "rather chooſe to be without them."

Tyr.————(*ſmiles.*)

Phi. What do you ſmile at, *Tyrunculus?*

Tyr. Nothing, Sir ———— I was only going to ſay I wiſh *Lucretius* liv'd near me.

Phi. That is not amiſs, *Tyrunculus*, I confeſs: However, I am as ready to ſerve you as he would be; and to ſhew you I am, if you approve of it, and have a Mind to have a little Touch at *Algebra*, I will give you the beſt Inſtructions I am capable of, upon Promiſe you will apply diligently; for were I ſure you would not, I ſhould repent my Folly even in aſking you, much more in the undertaking itſelf. You remember the old Proverb, *Strike the Iron while it is hot*: If you ſlight this Offer, you may, perhaps, afterwards blame yourſelf. ——— Come, what ſay you?

Tyr. Dear *Philomathes*, I am ſtill more obliged to you; and as I am bound in Duty to accept of your Kindneſs with Thanks, give me Leave to ſay, I will uſe my beſt Endeavours not to fruſtrate your kind Benevolence; but indeed it is giving you too much Trouble.

Phi. Do not mention it: You are welcome, as I ſaid before; but pray when will you begin, for the ſooner the better, if you take my Advice?

Tyr. That muſt be as you pleaſe, Sir.

Phi.

Phi. It never can be more convenient than now, as we are alone, and free from any Interruption.

Tyr. With all my Heart, Sir.

Phi. Well then, *Tyrunculus*, I would have you obſerve the Method I ſhall take for your Inſtruction: I ſhall firſt begin with you at *Vulgar Fractions*, (as you have, you ſay, but very little Notion of them) and ſhall treat more of them than is required in the *Algebraic* Part, that you may ſee their Uſe in other Reſpects: Then I ſhall proceed to *Algebraic Fractions*, the Rule of *Proportion*, and *Equations*; wherein I ſhall give you ſeveral Examples very rarely to be met with, or ſo eaſily demonſtrated: After theſe, I ſhall make ſome neceſſary Obſervations, and proceed directly to *Algebraic Problems*; and ſhall work them ſo gradually, that you cannot miſs (if you take any Pains at all) to underſtand every Operation. But as ſeveral Things will, no Doubt, happen, that you may not immediately, upon firſt reading, have a true Notion of; pray do ſo much Juſtice to yourſelf, as to aſk the Meaning of every Thing you are at a Loſs for, and do not content yourſelf to go away half taught. For my Part, I ſhall be careful to avoid any Thing that I think may give Occaſion to ſtop you in the Undertaking: Be you but as diligent to obſerve the Rules and Examples, and you will ſoon be Maſter of *that* which I ſhall hereafter inſtruct you in; and then *Tyrunculus*, *Cocker* and *Hammond* will at once appear both beautiful and eaſy to you; or *Saunderſon* and *Kerſey*, if you think fit to purchaſe them, (the former of which I more particularly recommend:) But if you have not the Happineſs to meet with either of theſe, you are here qualified for *Ward*, and have a good Foundation to peruſe more conciſe Authors, ſuch as *De Billy*, *Geſtinus*, *Wolfius*, &c. &c.

DIALOGUE II.

SECT. I.

On NOTATION *and* REDUCTION *of* VULGAR FRACTIONS.

Tyr. WHAT do you mean by *Notation of Fractions?*

Phi. Notation ſhews you how to write down and expreſs any *Fraction.*

Tyr. What is a *Vulgar Fraction?*

Phi. A *Fraction* ſignifies a broken Number, or, in other Words, when Unity, or the Number 1, is divided into any Number of Parts, thoſe Parts are a *Fractional* Part of the Integer itſelf, and is called a *Vulgar Fraction*

Tyr, How am I to know a *Vulgar Fraction?*

Phi. Whenever you ſee any Figure or Figures, with other Figure or Figures underneath, and a Daſh between them, (thus, $\frac{2}{3}$ $\frac{5}{4}$) they are *Vulgar Fractions* of ſome Denomination or other.

Tyr. What! Are there different Sorts then?

Phi. Yes, three at leaſt.

Tyr. Tell me their different Names, if you pleaſe?

Phi. Firſt then, there are *Simple*, *Single*, or *Proper Fractions*, (for you muſt note they are frequently called by either of theſe Names;) 2*dly*, *Improper*; and, 3*dly*, *Compound Fractions.*

Tyr.

Tyr. How are these seperately known, or expreſs'd in Figures ?

Phi. Thus ; $\frac{2}{3}$, $\frac{5}{8}$, $\frac{9}{14}$, $\frac{412}{716}$, *&c.* are all *Simple Fractions* ; they are ſo called becauſe each of the Numerators are leſs than the Denominators belonging to them.

Tyr. What do you mean by Numerator and Denominator ?

Phi. The Numerator always ſtand a-top of the Daſh, and the Denominator underneath : Thus, in the foregoing *Simple Fractions*, 2, 5, 9, and 412, are Numerators ; and 3, 8, 14, and 716, are their reſpective Denominators.

Tyr. Very well : What is an *Improper Fraction ?*

Phi. *Improper Fractions*, contrary to *Simple* ones, have their Numerators larger than the Denominators : Thus, $\frac{3}{2}$, $\frac{12}{7}$, $\frac{64}{9}$, and $\frac{716}{49}$, *&c.* are *Improper Fractions.*

Tyr. What do you mean by *Compound Fractions ?*

Phi. *Compound Fractions* are *Fractions* of *Fractions* compounded, coupled, or joined together, by the Word *of*, Thus, $\frac{2}{3}$ of $\frac{3}{4}$, or $\frac{5}{6}$ of $\frac{7}{11}$ of $\frac{21}{56}$, *&c.* are all *Compound Fractions :* Do you underſtand it ?

Tyr. Yes, ſurely : But how are theſe different *Fractions* read, or verbally expreſs'd ?

Phi. Thus, $\frac{2}{3}$ and $\frac{14}{29}$, is two Thirds and fourteen Twenty-ninths ; alſo $\frac{14}{5}$ is 14 Fifths, and $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{11}$, 3 Fourths of 5 Sixths of 7 Elevenths, *&c.*

Tyr. The 3 Fourths of the 5 Sixths of the 7 Elevenths : But of what ?

Phi. Why, the $\frac{3}{4}$ of the $\frac{5}{6}$ of the $\frac{7}{11}$ Parts of an Integer, or whole Number.

Tyr. I aſk Pardon : But this muſt be very hard to tell that.

Phi.

Phi. You are not to concern yourſelf about this at preſent; you will find it eaſy enough by-and-by.

Tyr. Are there no more Fractions?

Phi. Properly ſpeaking there are not; but there is what we call a *Mixt Number.*

Tyr. What is that pray?

Phi. A *Mixt Number* conſiſts of two Parts, the firſt Part a whole Number, and the other a *Fraction:* Thus, $4\frac{2}{9}$, and $247\frac{5}{8}$, are *Mixt Numbers*; that is, 4 whole Numbers and $\frac{2}{9}$ of Unity or 1, &c.

Tyr. I underſtand it very well: What is the next Thing you purpoſe?

Phi. Nothing more concerning the Names of *Fractions:* I ſhall now give you three or four Obſervations, which you will do well to remember.

OBSERV. 1.

The Value of every *Simple Fraction* is leſs than Unity or an Integer, by ſo many Times as the Numerator is contained in the Denominator; as you will ſee demonſtrated (*Caſe* 9) in *Reduction:* So alſo is the Value of all *Compound Fractions*, if they be compounded of *Simple* ones; for they are all but one *Simple Fraction* when reduc'd, as you will ſee (*Caſe* 6) in Reduction.

OBSERV. 2.

Contrary to theſe, the Value of any *Improper Fraction* is more than an Integer, or as many whole Integers as the Denominator is contained Times in the Numerator; (See *Caſe* 9 in *Reduction.*)

OBSERV. 3

When the Numerator and Denominator are alike, this is called by ſome an *Improper Fraction*, but with what

what Propriety I know not, ſeeing it is only Unity itſelf: For $\frac{4}{4}$ of a £. Sterling is 1 £. and $\frac{9}{9}$ of a Yard 1 Yard; becauſe 4, divided by 4, &c. make one whole Integer.

OBSERV. 4.

When you would make any whole Number into an *Improper Fraction*, then put Unity underneath it. Thus 4 will be $\frac{4}{1}$, and 126 will be $\frac{126}{1}$, &c. Pray remember this.

Tyr. I underſtand you quite well; but pray how is the Value of a *Fraction* diſcovered, in Order to know what Relation it bears to an Integer?

Phi. Fractions are reduced by certain Rules or Caſes in *Reduction*, of which if you be Maſter, you will ſoon *add*, *ſubtract*, *multiply*, and *divide*; but not elſe.

Tyr. Why is *Reduction* taught before the others pray?

Phi. Becauſe the *Fractions* muſt be firſt reduced before you can do the other Rules: *Reduction* therefore prepares the *Fractions* as you will ſee by the following Examples.

SECT. II.

REDUCTION *of* VULGAR FRACTIONS.

Tyr. I AM mightily pleaſed with what you have ſhewn me concerning the Nature of *Vulgar Fractions*; but I [illegible] long to know how to reduce them.

Phi. That you ſhall directly; but are you ſure you know what a *Simple*, *Compound*, *Improper Fraction*, and *Mixt Numbers* are? for they muſt be known: And if you think you do not underſtand them quite well, give them another Look; you cannot be too perfect.

Tyr. I am poſitive I underſtand what they mean.

Phi. Very well: Pray hand me that Slate then; I will reduce them in their Order before your Face, and you may try at other Examples, which you may ſet yourſelf at your Leiſure.

CASE I.

To reduce a Mixt Number *to an* Improper Fraction.

The Rule is,

Multiply the whole Number by the Denominator of the *Fraction* belonging to it, and take in the Numerator; then under this Product ſet the Denominator; ſo is this *Improper Fraction* equivalent to to the *Mixt Number* given.

EXAMPLE 1.

Reduce $4\frac{2}{5}$ *to an* Improper Fraction.

$$\begin{array}{r} 4\frac{2}{5} \\ 5 \\ \hline 22 \end{array}$$

$\frac{22}{5}$ *Ans.*

EXAMPLE 2.

Reduce $51\frac{2}{11}$ *to an* Improper Fraction.

$$\begin{array}{r} 51\frac{2}{11} \\ 11 \\ \hline 563 \end{array}$$

$\frac{563}{11}$ *Ans.*

EXAMPLE 3.

Reduce $576\frac{14}{24}$ *to an* Improper Fraction

$$\begin{array}{r} 576\frac{14}{24} \\ 24 \\ \hline 2308 \\ 1153 \\ \hline 13838 \end{array}$$

$\frac{13838}{24}$ *Ans.*

Tyr.

Tyr. He that can do common Multiplication may do this.

Phi. True; and he that can do common Division may do the next, it being only the Reverſe of the former Caſe, as you'll ſee by the ſame Examples.

CASE 2.

To reduce an Improper Fraction *to its* equivalent, whole, *or* mixt Number.

Rule is,

Divide the Numerator by the Denominator, and if any Thing remains, ſet it over the Denominator, for a new Numerator, and it is done.

EXAMPLE 1.

Reduce $\frac{22}{5}$ *to its* equivalent, whole, *or* mixt Number.

$$5)\ 22$$

$$4\tfrac{2}{5}$$ *Anſ.* *See Ex.* 1. *Caſe* 1.

EXAMPLE 2.

Reduce $\frac{563}{11}$ *to its* equivalent, whole, *or* mixt Number.

$$11)\ 563$$

$$51\tfrac{2}{11}$$ *Anſ.*

EXAMPLE 3.

Reduce $\frac{13838}{24}$ *to its* equivalent, whole, *or* mixt Number.

$$24)\,13838\,(576\tfrac{14}{24}\ \textit{Anf.}$$

```
24)13838 (576 14/24 Anſ.
   120
   ---
    183
    168
    ---
     158
     144
     ---
      14
     ---
```

Note, When there is no Remainder in the Diviſion, then will the whole Number be equivalent, or equal to the given *Improper Fraction*. As for Example: Suppoſe I would reduce $\frac{56}{7}$ to its Equivalent, I divide 56 by 7, and the Quotient is 8; ſo is 8 equal to $\frac{56}{7}$; ſo alſo $\frac{228}{82}$ is equal to 19. This is eaſily ſeen by the next Caſe.

$3\frac{42}{82}$ appears wrong

CASE 3.

To reduce any whole Number *to an* Improper Fraction.

Rule is,

Multiply the whole Number by any Figure at Pleaſure, and under the Product ſet the ſame Figure you multiply'd by, and you have an *Improper Fraction* equal to the given whole Number.

Ex-

EXAMPLE 1.

Reduce 14 *to an* Improper Fraction.

$$\begin{array}{ccc} 14 & 14 & 14 \\ 5 & 9 & 12 \\ \hline 70 & 126 & 168 \\ \text{— } Anſ. & \text{— } Anſ. & \text{— } Anſ. \\ 5 & 9 & 12 \end{array}$$

Here you ſee I multiply the whole Number by 5, by 9, or by 12, or any other Figure, and the *Improper Fractions* are all equal to each other, and are alſo equal to 14. Therefore this is an unlimited Queſtion, to which an infinite Number of Anſwers may be given, and all right ; but if the Queſtion be propoſed thus, it will be limited, and can then have only one Anſwer. As for Example.

EXAMPLE 2.

Reduce 14 *to an* Improper Fraction, *whoſe Denominator ſhall be* 15.

Here as I am to have 15 for its Denominator, I am oblig'd to multiply by 15, and no other Figure.

$$\begin{array}{r} 14 \\ 15 \\ \hline 70 \\ 14 \\ \hline 210 \end{array}$$

— *Anſ. equal to* 14, *as before.*

15

CASE 4.

To reduce a Fraction *to its loweſt Terms, equal in* Value *to the* Fraction *given.*

Rule is,

Divide the Numerator and Denominator by any Figure that will divide them both without any Remainder, and continue ſo doing till you can divide them no lower; ſo will this laſt Quotient be the loweſt Terms equal to the original given *Fraction.*

EXAMPLE 1.

Reduce $\frac{144}{216}$ *to its loweſt Terms.*

Diviſors	2	3	4	3	
Num.	144	72	24	6	2
Denom.	216	108	36	9	3

Anſ. $\frac{2}{3} = \frac{144}{216}$

EXAMPLE 2.

Reduce $\frac{576}{960}$ *to its loweſt Terms.*

Diviſors	6	8	4	
Num.	576	96	12	3
Denom.	960	160	20	5

Anſ. $= \frac{567}{906}$

Do you underſtand the Work?

Tyr. I underſtand all very well, but the two Lines you make after the Anſwers I don't rightly apprehend.

Phi.

Phi. What this Mark (=) do you mean ?

Tyr. Yes.

What = signifies. *Phi.* It is the Sign of Equality, it ſignifies that $\frac{2}{3}$ is equal to $\frac{144}{216}$; and $\frac{3}{5}$ is equal to $\frac{576}{960}$. You will frequently ſee it uſed by and by.

Note, After the ſame Manner are *Algebraic Fractions* abbreviated. For ſuppoſe I were to abbreviate or reduce $\frac{abb}{bc}$ to its loweſt Terms; it is only taking away ſuch Letters or Quantities as are alike out of the Numerator and the Denominator, and the Work is done. Thus in the above *Algebraic Fraction* $\frac{abb}{bc}$, I find b both in the Numerator and Denominator, therefore by taking b from both, I have in $\frac{ab}{c}$ its loweſt Terms $= \frac{abb}{bc}$. But this you will ſee more of in Caſe 4 of *Algebraic Fractions.*

Tyr. Is there no other Method of reducing a *Fraction* to its loweſt Terms, becauſe it is difficult to find Figures that will divide ſome *Fractions.*

Phi. 'Tis true, for that Figure which will divide one, will not perhaps divide the other, therefore there is a Way to tell what Figure will do it at one Operation.

Tyr. That muſt be mighty pretty, pray let's ſee it?

Phi. You ſhall.

CASE

CASE 5.

Another Way to reduce a Fraction *to its loweſt Terms at one Work.* (Euc. 7, *Pr.* 1, 2, 3.)

Divide the Denominator of the *Fraction* by its Numerator, and if any Thing remains, divide your former Diviſor by it, and if any Thing yet remains, divide your laſt Diviſor by *that*; thus proceed till you have Nothing remain, and then ſhall your laſt Diviſor be a *Common-Meaſurer*, that will infallibly divide both the Numerator and Denominator of the given *Fraction* into its loweſt Terms at one Work.

Tyr. Pray give me an Example, and explain it in Words.

Phi. I will.

EXAMPLE I.

Reduce $\frac{147}{252}$ *to its loweſt Terms by a* Common-Meaſurer.

Firſt, I divide 252 by 147 the Numerator, and it goes once, and 105 remains over; this 105 I make now a Diviſor, and the laſt Diviſor (*viz.* 147) a Dividend, and find it contains once 105, and 42 remains over; by this 42 I divide the laſt Diviſor 105, and find 21 remains; and laſtly, by this 21 I divide the laſt Diviſor 42, and find Nothing remains over: So is the laſt Diviſor 21 a *Common-Meaſurer*, that will reduce the given *Fraction* $\frac{147}{252}$ to its loweſt Terms at once; for dividing the *Numerator* 147 by 21, I have 7 for a new Numerator; and dividing 252 by 21, I have 12 for a new Denominator; and thus I find $\frac{7}{12} = \frac{147}{252}$.

Ex-

EXAMPLE 2.

Reduce $\frac{574}{861}$ *to its loweſt Terms by a* Common-Meaſurer.

```
           574) 861 (1
                574
                ---
Com. Meaſure 287) 574 (2
                  574
                  ---
                   0

         Num.
     287) 574 (2 New Num.
          574
          ---
           0

        Denom.
     287) 861 (3 New Denom.
          861
          ---
           0
```

Anſ. $\frac{2}{3} = \frac{574}{861}$.

Tyr. I underſtand it well; but pray ſuppoſe a *Fraction* cannot be abbreviated by a *Common-Meaſurer*, by its proving an Unit at laſt?

Phi. Why then it is in the loweſt Terms already: Such a one is $\frac{176}{547}$.

Tyr. Very well. Pray are there no more Caſes in Reduction?

Phi. Yes, here follows,

CASE

CASE 6.

To reduce a Compound Fraction *to a* Simple-one *of the ſame Value.*

Rule is,

Multiply all the Numerators one into another for a new Numerator ; then multiply all the Denominators together for a new Denominator, ſo ſhall this new *Fraction* be equal to the *Compound Fraction* given.

Tyr. This is ſo eaſy I think I can do it directly ; pray try me ?

Phi. No Doubt, for it is only common Multiplication.

EXAMPLE I.

Reduce $\frac{2}{3}$ *of* $\frac{5}{6}$ *of* $\frac{3}{4}$ *to a* Simple Fraction.

Tyr. I ſet down all the Numerators thus,

		2
		5
Then I multiply all the Denom.	3	10
	6	3
	18	30 New N.
	4	
New Denom.	72	

Anſ. $\frac{30}{72} = \frac{2}{3}$ of $\frac{5}{6}$ of $\frac{3}{4}$.

Phi.

What this × Crofs fignifies.

Phi. It is very right, *Tyrunculus* ; but there is a Character ufed for Multiplication which will mightily fhorten the Work, and take up lefs Room ; befides it is frequently ufed in *Algebra.* This is it (×,) and it fignifies that all the Numbers between which it ftands are to be multiplied together. Thus $4 \times 6 \times 2 = 48$. Pray remember it.

Tyr. I know your Meaning immediately. Thus 4 multiplied by 8 is 4×8, that is 32. So $3 \times 2 \times 5 = 30$. Is it fo or not?

Phi. You are very right ; now I'll try you with another Sum.

EXAMPLE 2.

Reduce $\frac{5}{7}$ *of* $\frac{4}{5}$ *of* $\frac{3}{4}$ *of* $\frac{11}{12}$ *to a* Simple Fraction.

Tyr. I multiply all the Numerators together,

Thus $5 \times 4 \times 3 \times 11 = 660$ for a new Numerator.

And $7 \times 5 \times 4 \times 12 = 1680$ for a new Denominator.

So is $\frac{660}{1680} = \frac{5}{7}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{11}{12}$ *Anf.*

Phi. It is very well done *Tyrunculus*, now we will proceed to

CASE 7.

To reduce Fractions *of unequal, or different Denominators, to* Fractions *of the fame Value, having but one common Denominator to all the Numerators.*

Rule is,

Multiply all the Denominators together for a common Denominator ; then take each Numerator, (beginning

ginning at the firſt) and multiply it into all the Denominators except its own Denominator; ſo ſhall theſe different Products be new Numerators to the common Denominator, equal to that *Fraction* whoſe Numerator you multiplied into the Denominators, which placed over the common Denominator, and the Work is done. Do you think you could do this directly?

Tyr. No, this is not ſo eaſy as the laſt Caſe. Pray give me one Example at large, and then I'll try?

What N. N. and C. D. ſignify.

Phi. I will. Pray remember N. N. ſignify new Numerator, and C. D. common Denominator.

EXAMPLE I.

Reduce $\frac{2}{3}$, $\frac{3}{5}$, *and* $\frac{5}{8}$, *to* Fractions *of the ſame Value, having a common Denominator.*

$\frac{2}{5}$, $\frac{3}{5}$, $\frac{5}{8}$.

Firſt 3	then 2	and 3	and 5
5	5	3	5
15	10	9	25
8	8	8	3
C.D. 120	N. N. 80	N. N. 72	N. N. 75

Anſ. $\frac{80}{120} = \frac{2}{3}$, *for* 80 *is* $\frac{2}{3}$ *of* 120. *Alſo* $\frac{72}{120} = \frac{3}{5}$, *and* $\frac{75}{120} = \frac{5}{8}$.

3
24 3

Tyr. 'Tis ſo plainly done, that I think I can do another Example.

Phi. Poſſibly you may, as you take good Obſervation.

Ex-

EXAMPLE 2.

Reduce $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, *and* $\frac{3}{4}$, *to* Fractions *having a common Denominator.*

Tyr. Firſt then, $5 \times 6 \times 8 \times 4 = 960$ for a common Denominator. Then $4 \times 6 \times 8 \times 4 = 768$ N. N. And $5 \times 5 \times 8 \times 4 = 800$ N. N. Again, $7 \times 6 \times 5 \times 4 = 840$ N. N. And laſtly, $3 \times 8 \times 6 \times 5 = 720$ N. N. Theſe new Numerators plac'd over the common Denominator, I find the Anſwer to be $\frac{768}{960} = \frac{4}{5}$, $\frac{800}{960} = \frac{5}{6}$, $\frac{840}{960} = \frac{7}{8}$, and $\frac{720}{960} = \frac{3}{4}$. Is it right?

Phi. You ſurprize me, to ſee you ſo apt; ſee what Care is! You have no Occaſion for more Examples in this Caſe. We will paſs on then to

CASE 8.

To reduce Fractions *of one Denomination to another.*

This conſiſts of two Parts, *aſcending* or *deſcending*. And firſt of *aſcending*.

When a *Fraction* is given to be brought from a leſs to a greater Denomination, then ſet down the *Fraction*, and make a compound one of it, according to the Denomination it is to be brought into; and this *compound Fraction* is made by conſidering how many of the leſs make one of the greater; then reduce this *compound* to a *ſimple Fraction*, and you will have a *Fraction* of another Denomination, equal in Value to the given *Fraction*.

Tyr. This is a hard Caſe, this is not underſtood by bare reading.

Phi. It is harder than ſome of the reſt; but an Example or two will make it plain.

EXAMPLE 1.

Reduce $\frac{3}{5}$ *of a Penny to the* Fraction *of a* £. *Sterling.*

Now obſerve, as 12 Pence make a Shilling, and 20 Shillings a Pound, I make a *compound Fraction* of $\frac{3}{5}$ thus,

$\frac{3}{5}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a £.

Now this reduced to a *ſimple Fraction, viz.* $3 \times 1 \times 1 = 3$ N. N. and $5 \times 12 \times 20 = 1200$ N. D. So is $\frac{3}{1200}$ of a £. $=$ $\frac{3}{5}$ of a Penny.

OR,

Otherwiſe make a *compound Fraction* of it at once; that is, 240 Pence make a £. Sterling. Then it will be $\frac{3}{5}$ of $\frac{1}{240}$; this reduced to a *ſimple Fraction*, is $\frac{3}{1200}$ of a £. $=$ $\frac{3}{5}$ of a Penny as above.

EXAMPLE 2.

Reduce $\frac{3}{4}$ *of a Farthing to the* Fraction *of a Guinea.*

This will be $\frac{3}{4}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{21}$ of a Guinea.

Now $3 \times 1 \times 1 \times 1 = 3$ N. N. and $4 \times 4 \times 12 \times 21 = 4032$ N. D. So is $\frac{3}{4032}$ of a Guinea $=$ $\frac{3}{4}$ of a Farthing.

2. DESCENDING.

In *deſcending* you are to conſider, that the *Fraction* is to be brought from a greater to a leſs Denomination; therefore, as you multiplied the Denominator of the *given Fraction* by the Parts contained in the Integer in Reduction *aſcending*, ſo now here you muſt multiply the Numerator of the *given Fraction* by the ſame

ſame Parts, and you have the Anſwer. Or, which is all the ſame, only invert the Parts contained in the Integer, (that is, turn them topſy-turvy) and make of them a *compound Fraction* as before, then reduce it to a *ſimple Fraction*, and it is done.

EXAMPLE 1.

Reduce $\frac{4}{5}$ *of a £. Sterling to the* Fraction *of a Penny.*

Here I conſider that a Shilling is $\frac{1}{20}$ of a £. and a Penny $\frac{1}{12}$ of a Shilling, therefore I multiply the Numerator 4 by 20 and by 12, and the Product is 960, which I place over the Denominator, thus $\frac{960}{5}$. So is $\frac{960}{5}$ of a Penny equal to $\frac{4}{5}$ of a £. or 16 Shillings.

Or, by inverting the Parts as above directed, it will be $\frac{4}{5}$ of $\frac{20}{1}$ of $\frac{12}{1}$ a *compound Fraction*, which reduced to a *ſimple one*, *viz.* $4 \times 20 \times 12 = 960$ N. N. and $5 \times 1 \times 1 = 5$ N. D. To prove this we will try *Example* 2. of Reduction *aſcending*.

EXAMPLE 2.

Reduce $\frac{3}{4032}$ *of a Guinea to the* Fraction *of a Farthing.*

Here $\frac{3}{4032}$ of $\frac{21}{1}$ of $\frac{12}{1}$ of $\frac{4}{1}$.
Now $3 \times 21 \times 12 \times 4 = 3024$ N. N.

And $4032 \times 1 \times 1 \times 1 = 4032$ N. D. This *Fraction* abbreviated is $= \frac{3}{4}$ of a Farthing. So is $\frac{3}{4}$ of a Farthing $= \frac{3}{4032}$ of a Guinea, as in *Example* 2 of laſt Rule.

What do you ſay to this Caſe, *Tyrunculus?*

Tyr. I think I underſtand it pretty well; however, I will look it over again, and try at other Examples.

Phi. Do ſo. Now, *Tyrunculus*, we are come to the

 moſt

moſt uſeful and pleaſant Caſe of all, which is to find the true Value of any *Fraction.*

Tyr. That I ſhall like I know.

Phi. Pray obſerve carefully the Rules and Examples, and I dare ſay you will work any of them directly after me.

CASE 9.

To find the Value of a Fraction *in Money, Weight, or Meaſure.*

Rule is,

Multiply the Numerator by the Parts contain'd in the Integer to which it belongs, always obſerving to begin with that Part neareſt related to the Integer; then divide by the Denominator, and if any Thing remains, multiply it by the next greateſt Part neareſt related to the Integer, and divide again by the Denominator. Thus proceed till you can reduce it no lower for Want of Parts in the Integer, and the Work is done.

Tyr. I muſt beg one Example at large.

Phi. You ſhall, and you will need no more to underſtand the Caſe.

EXAMPLE I.

What is the Value of $\frac{3}{32}$ *of a £. Sterling?*

Firſt, in Order to find the Value of this *Fraction,* I conſider the next neareſt Part related to a £. and I find it to be Shillings. Now becauſe 20 Shillings make a £. I multiply the Numerator 3 by 20, and it is 60, which I divide by the Denominator 32, and have 1 in the Quotient, which is 1 Shilling, and 28 remains over; this I call $\frac{28}{32}$ of a Shilling. Now as 12 Pence make a Shilling, multiply 28 the Numerator

rator by 12, and it makes 336, which I divide alſo by the Denominator 32, and the Quotient is 10, which is 10 Pence, and 16 remains over; this I call $\frac{16}{32}$ of a Penny; then as 4 Farthings make a Penny, I multiply the Numerator 16 by 4, and it is 64, which I divide again by the Denominator 32, and the Quotient is 2 Farthings, So that I find $\frac{3}{32}$ of a £. to be 1 *s.* 10 *d.* $\frac{1}{2}$. *See the Work.*

```
        3
       20
       --
  32) 60 (1 s
      32
      --
      28
      12
      --
 32) 336 (10 d.
     32
     --
      16
       4
      --
  32) 64 (2 qrs.
      64
      --
       0
```

Anſ. 1 *s.* 10 *d.* $\frac{1}{2}$.

What is the $\frac{4}{25}$ of a Moidore?

```
       4
      27
     ---
25) 108 (4 s.
    100
     --
       8
      12
      --  Ans. 4 s. 3 d. 3/4 9/25.
25)  96 (3 d.
     75
     --
     21
      4
     --
25)  84 (3 qrs.
     75
     --
      9
```

Questions to be tried.

3. What's the Value of $\frac{4}{12}$ of an Cwt. } *Anſ.* 1 *qr.* 9 ℔ 5 *oz.* 5 *drs.* $\frac{4}{12}$

4. What's the $\frac{127}{317}$ of a Year? } *Anſ.* 146 *d.* 5 *h.* 31 *m.* 36 *ſ.* $\frac{168}{317}$

☞ *To find the Value another Way.*

Suppoſe I wanted to know the $\frac{5}{12}$ of a £. or the $\frac{7}{18}$ of a Moidore.

This

This is done only by abbreviating the Numerator, and bringing it down to Unity; then will the *Fraction* be $\frac{1}{12}$ of a £. then multiply the Value by the Numerator, and you have its true Value.

First, What is the $\frac{5}{12}$ *of a £. Sterling?*

I abbreviate the Numerator to Unity or 1, and considering what $\frac{1}{12}$ of a £. is, I find it to be 1 *s.* 8 *d.* this I multiply by 5 the Numerator, and it makes 8 *s.* 4 *d.* which is the Value of $\frac{5}{12}$ of a £. Sterling. Again,

What is the $\frac{7}{18}$ *of a Moidore?*

I consider that $\frac{1}{18}$ of a Moidore is 1 *s.* 6 *d.* then by multiplying by the Numerator 7, I have 10 *s.* 6 *d.* which is the Value of $\frac{7}{18}$ of a Moidore.

N. B. I told you in *Dial.* the 2*d*, *Observ.* 1, that every *simple Fraction*'s Value was less than an Integer or Unity, and that the Value of an *improper Fraction* was more. (*Observ.* 2.) To prove which let us take any two *Fractions*, one *simple*, and the other *improper*, and see what their Value is in Relation to a £. Sterling.

EXAMPLE I.

Suppose the Simple Fraction *to be* $\frac{9}{12}$ of a £.

I find $\frac{1}{12}$ of a £. to be 1 *s.* 8 *d.* therefore $\frac{9}{12}$ is 9 Times 1 *s.* 8 *d.* = 15 *s.* Now 15 *s.* wants 5 *s.* of the whole Integer or 1 £. therefore the *simple Fraction* $\frac{9}{12}$ is less in Value than the whole Integer or Unity, by 5 Shillings. Now contrary to this, an im-

improper Fraction's Value is more than the Integer itſelf; and its Value is greater or leſs according to the Largeneſs of its Numerator.

EXAMPLE 2.

What is the $\frac{25}{4}$ of a £. Sterling?

This being an *improper Fraction*, I divide the Numerator 25 by the Denominator 4, and it gives 6 whole Integers, or 6 £. now as one remains over, it is $\frac{1}{4}$ of a £. which is 5 *s.* So I find $\frac{25}{4}$ of a £. to be 6 £. 5 *s.* 10 *d.* Do you underſtand it, *Tyrunculus?*

Tyr. Had you only ſaid it I might have been at a Loſs; but you have demonſtrated it ſo plainly, that I muſt be quite dull of Apprehenſion not to ſee the Nature of it.

Phi. I am glad you underſtand me; and pray do you think you underſtand all the 9 Caſes in *Reduction* ſo as to work them now off Hand; for if you do not (at leaſt all but the 8*th*, that being not ſo much wanted) I freely tell you that you will be at a great Loſs; for the next four Rules depend wholly upon a true Knowledge of *Reduction.*

Tyr. You do well, *Philomathes*, to take ſuch Care of me, and I hope every Learner will take your Advice; but for my own Part, I can ſafely ſay I underſtand *Reduction* quite perfectly.

Phi. Well, if ſo, we will proceed directly to *Addition.*

DIA-

DIALOGUE III.

Of ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION, of VULGAR FRACTIONS.

SECT. I.

Of ADDITION of VULGAR FRACTIONS.

Tyr. HOW is *Addition* of *Fractions* performed?

Phi. By this one general Rule, *viz.* all *compound Fractions* muſt firſt be reduced to *ſimple ones*, and all *Fractions* to a common Denominator, (by *Caſe* the 6*th* and 7*th* of *Reduction*;) then add all the Numerators together as in common *Addition*, and place their Sum over the common Denominator; and if it be an *improper Fraction*, reduce it to a mixt Number (by *Caſe* 2. in *Reduction*) and you have the Sum of all the *Fractions*.

EXAMPLE 1.

Add $\frac{2}{5}$, $\frac{1}{5}$, *and* $\frac{4}{5}$ *together.*

Here becauſe the *Fractions* have all one common Denominator, I only add the Numerators 2, 1, and 4 together, and their Sum is 7, which I place over the common Denominator 5, and the Sum is $\frac{7}{5}$ an *improper Fraction*, equal to 1 $\frac{2}{5}$ *Anſ.*

EXAMPLE 2.

Add $\frac{5}{21}$, $\frac{6}{21}$, $\frac{14}{21}$ *and* $\frac{19}{21}$ *together.*

Anſ. $\frac{44}{21} = 2\,\frac{2}{21}$.

Tyr.

Tyr. This is mighty eafy; the Rule is fo plain one cannot well mifs.

Phi. Now I will fet you a Queftion, *Tyrunculus.*

EXAMPLE 3.

Add $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, *and* $\frac{5}{6}$ *together.*

Tyr. I am afraid you have puzzled me; but ftay a little, let me find it out myfelf.—I fee how it muft be done, I muft reduce the *Fractions* firft to a common Denominator, and then add the Numerators together as you have done above. Muft I not?

Phi. You have it I perceive.

Tyr. Firft then, to reduce the *Fractions* to a common Denominator.

$\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$

I multiply $3 \times 2 \times 4 \times 6 = 144$ C. D. Then $2 \times 2 \times 4 \times 6 = 96$ N. N. Then $1 \times 3 \times 4 \times 6 = 72$ N. N. Again, $3 \times 2 \times 3 \times 6 = 108$ N. N. Laftly, $5 \times 4 \times 2 \times 3 = 120$. So that I find the new Numerators are as follows:

	N. Numerators.
	96
C. D. 144	72
	108 and
	120
Their Sum	396 which placed over the

common Denominator 144, ftands thus, $\frac{396}{144}$; this, by *Cafe* the 2*d.* in *Reduction*, $= 2\frac{108}{144}$, that is, $2\frac{3}{4}$, the Sum of $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$.

Phi. It is quite right, *Tyrunculus*, you fee what it is to mind the Rules given for Inftruction; you will

will do *that* already which will coſt a careleſs Reader ten Times the Trouble. Come, I'll try you with another.

Tyr. With all my Heart.

EXAMPLE 4.

Phi. *Add* $\frac{2}{4}$ *of* $\frac{4}{5}$ *of* $\frac{3}{4}$ *and* $\frac{4}{6}$ *of* $\frac{5}{8}$ *together.*

Try. Let me ſee: I muſt firſt (by *Caſe* 6. of *Reduction*) reduce the *compound* to *ſimple Fractions*, and then the *Fractions* to a common Denominator, and proceed as before. Firſt then I multiply all the Numerators together, *viz.* $2 \times 4 \times 3 = 24$ N. N. Then $4 \times 5 \times 4 = 80$ C. D. So is $\frac{24}{80} = \frac{2}{4}$ of $\frac{4}{5}$ of $\frac{3}{4}$. Then the other *compound Fraction, viz.* $\frac{4}{6}$ of $\frac{5}{8}$ reduced, is $= \frac{20}{48}$. So the two *ſimple Fractions* to be added are $\frac{24}{80}$ and $\frac{20}{48}$. Theſe I reduce to a common Denominator, as in *Example* 3, and find them to be $\frac{1152}{3840}$ and $\frac{1600}{3840}$. Then have I nothing to do but add the Numerators together, and their Sum is 2752, which I place over the common Denominator thus, $\frac{2752}{3840}$, and this I apprehend to be the Sum of $\frac{2}{4}$ of $\frac{4}{5}$ of $\frac{3}{4}$ and $\frac{4}{6}$ of $\frac{5}{8}$. Is it ſo or not?

Phi. It is, and I am proud to ſee you ſo well grounded in the Rules of Reduction: However, I think I can poſe you the very next Queſtion. Will you try at it?

Tyr. To be ſure I will; for I imagine you will ſhew me if I cannot do it.

Phi. You need not doubt it—Come then.

EXAMPLE 5.

Add 4 £. $\frac{3}{4}$, 7 £. $\frac{3}{5}$, *of* $\frac{2}{3}$ *and* $\frac{5}{12}$ *together.*

Tyr. ——————————

Phi. I thought I ſhould puzzle you; however, be not diſcouraged.

Tyr.

Tyr. There is Nothing that puzzles me but the laſt *Fraction* $\frac{5}{12}$, for I am not certain whether it belongs to the *compound Fraction* that ſtands before it, or whether it be a ſeparate *Fraction* by itſelf.

Phi. It cannot belong to the *compound Fraction*, becauſe the Word *of* is not between them. There are two Ways to do this and ſuch like Queſtions; but the ſecond is the ſhorteſt and eaſieſt Method in my Opinion. I ſhall therefore only tell you the Way to work it, and leave you to try it by yourſelf at large*. Obſerve then,

METHOD I.

Reduce the *compound Fraction* $\frac{3}{5}$ of $\frac{2}{3}$ to a *ſimple-one*, which is $\frac{6}{15}$. Then will the Sum be thus, Add 4 £. $\frac{3}{4}$, 7 £. $\frac{6}{15}$ and $\frac{5}{12}$ together. Now by reducing the *mixt Numbers* 4 $\frac{3}{4}$ and 7 $\frac{6}{15}$ into *improper Fractions*, I have $\frac{19}{4}$ and $\frac{111}{15}$. Then may the Sum be read thus; add $\frac{19}{4}$, $\frac{111}{15}$ and $\frac{5}{12}$ together. Theſe *Fractions* I reduce to a common Denominator (as in *Example* 3.) and find them to be $\frac{3420}{720}$, $\frac{5328}{720}$ and $\frac{300}{720}$. Theſe Numerators added together, and divided by the common Denominator 720, gives 12 in the Quotient, and 408 remaining over. So is the Anſwer 12 £. $\frac{408}{729}$ = 12 £. 11*s*. 4*d*.

METHOD 2.

The ſecond Way certainly is the beſt, becauſe you have no Buſineſs to meddle with the whole Numbers, but only with the *Fractions*, and then add their Sum to the whole Numbers afterwards.

The

* It is ſuppoſed, that the Learner by this Time knows how to reduce mixt Numbers to *improper Fractions*, and *Compound* to *Simple-ones*, having had ſo many Examples of both Kinds.

The *Fractions* are $\frac{3}{4}$, $\frac{3}{5}$ of $\frac{2}{3}$ and $\frac{5}{12}$. Now $\frac{3}{5}$ of $\frac{2}{3}$ $= \frac{6}{15}$. Therefore add $\frac{3}{4}$, $\frac{6}{15}$ and $\frac{5}{12}$ together.

These *Fractions* reduced to a common Denominator will be $\frac{540}{720}$, $\frac{288}{720}$ and $\frac{300}{720}$. These Numerators added together, and divided by the common Denominator 720, gives 1 whole Number in the Quotient, and 408 remains over. So is the Sum of the *Fractions* only 1 $\frac{408}{720}$, which I add to the whole Numbers as follows, and have the same Answer as above.

Add 4 £.
7
1 $\frac{408}{720}$

Ans. 12 $\frac{408}{720}$ *as before* = 12 £. 11 *s.* 4 *d.*

Tyr. This Way is the best I see; because if the whole Numbers consist of many Places of Figures, then by the first Method in reducing them to *improper Fractions*, there will be a great many Figures, and a great deal of Work to reduce them to a common Denominator afterwards; whereas, if the whole Numbers be ever so large, it makes no Alteration in this second Way of doing it.

Phi. Your Notion is right, *Tyrunculus*; and I think you will soon learn to *subtract*, you are so perfect in *Addition*.

Tyr. I will do the best I can.

Phi. Well, *Tyrunculus*, who can desire any further? You have done well hitherto, and I hope will continue it. We will proceed then to.

SECT. II.

Of SUBTRACTION *of* VULGAR FRACTIONS.

Tyr. I Have heard that *Subtraction* is the hardeſt Rule in *Fractions.*

Phi. It is counted ſo by moſt Learners; however, he that underſtands *Reduction* well may ſoon do it, as you will preſently find.

Tyr. Pray how is it performed?

Phi. The very ſame as *Addition*, ſave only you are to ſubtract inſtead of adding, according to the Nature of the Queſtion; but the Rule is the ſame. For all *compound Fractions* muſt be reduced to *ſimple-ones*, and then all to a common Denominator; after which only take the Numerator of the *Fraction* to be ſubtracted, out of the Numerator of the other *Fraction*, and you have the Difference or Anſwer as in common *Subtraction.*

Ex. 1.	Ex. 2.	Ex. 3.
From $\frac{19}{64}$	From $\frac{14}{19}$	From $\frac{136}{144}$
Take $\frac{11}{64}$	Take $\frac{11}{19}$	Take $\frac{98}{144}$
Anſ. $\frac{8}{64}$	*Anſ.* $\frac{3}{19}$	*Anſ.* $\frac{38}{144}$
Proof. $\frac{19}{64}$	*Proof.* $\frac{14}{19}$	*Proof.* $\frac{136}{144}$.

Here in theſe three Examples, becauſe the *Fractions* have a common Denominator, I only ſubtract the Numerators as in common *Subtraction*, and place the Difference over the common Denominator for an Anſwer. I prove the Work alſo as in common *Subtraction*; for I add the Numerator of the Difference to the Numerator of the leſs *Fraction*, which, if the Work

Work be right, will be equal to the Numerator of the greater *Fraction*.

Tyr. I think this is more diverting than *Addition*. Now let me aſk you a Queſtion or two if you pleaſe.

EXAMPLE 4.

From $\frac{13}{15}$ *take* $\frac{4}{11}$.

Phi. Firſt, I reduce the *Fractions* to a common Denominator, and find them $\frac{143}{165}$ and $\frac{60}{165}$, then I ſubtract the Numerator 60 from the Numerator 143, and there remains 83; which placed over the common Denominator 165, gives $\frac{83}{165}$ for the Difference.

EXAMPLE 5.

From $\frac{4}{6}$ *of* $\frac{7}{8}$ *take* $\frac{2}{5}$ *of* $\frac{1}{3}$

That is, from $\frac{28}{48}$ take $\frac{2}{15}$. Theſe reduced to a common Denominator, it will be, From $\frac{420}{720}$ take $\frac{96}{720}$. Now theſe are prepared for Work; therefore, by ſubtracting 96 from 420, I have 324 remaining: So is $\frac{324}{720}$ the Difference between $\frac{4}{6}$ of $\frac{7}{8}$ and $\frac{2}{5}$ of $\frac{1}{3}$.

Tyr. There can be Nothing eaſier than theſe Examples; he that underſtands *Addition* cannot miſs *Subtraction* indeed. But ſtill I have taken Notice of one Thing, which perhaps if I mention you will laugh at me.

Phi. Why ſhould you think ſo; *that* would be highly baſe in me, when I have before deſired you to aſk me any Thing that you are doubtful of; therefore pray let's hear it, it may perhaps be of more Service than you are aware of?

Tyr. It is this then: In all the foregoing Examples I perceive that the Numerator of the *Fraction* to be ſubtracted is leſs than the Numerator of the *Fraction* you ſubtract from, which makes all the Examples

 quite

quite eaſy: But ſuppoſe the Numerator of the *Fraction* to be ſubtracted be larger than the other *Fraction*, where can I take it out of then, and how muſt I proceed in ſuch a Caſe?

Phi. You were afraid I ſhould laugh at you, but I aſſure you it is a very material Queſtion, for this is the moſt difficult Part of *Subtraction* that Learners meet with. The Rule then is this:

N. B. When the Numerator of the lower *Fraction* (that is, the *Fraction* to be ſubtracted) is larger than the *Fraction* you ſubtract from, then take the ſaid Numerator out of the common Denominator, and to that Difference add the top, or leſs Numerator, ſo ſhall this be a new Numerator to be placed over the common Denominator, and you muſt carry one for borrowing out of the common Denominator, as you do when you borrow in common *Subtraction*, when the lower Figure is larger than the Top-one.

Tyr. This is quite plain, and eaſy enough to be performed I ſhould think.

Phi. Eaſy; can a Thing be hard, when the Rule laid down to work it by tells you how to proceed in every Reſpect? However, I will try you with a Queſtion.

EXAMPLE 6.

From 24 £. $\frac{5}{10}$, *take* 19 £. $\frac{6}{10}$.

Tyr. I try to ſubtract the Numerator 6 from the Numerator 5, but cannot; therefore by the Rule I take 6 out of the common Denominator 10, and there remains 4, to which I add the leſs Numerator 5, and that makes 9, which I place over the common Denominator 10, and it is $\frac{9}{10}$. Then becauſe I borrowed out of the common Denomination 10, I carry 1 to the whole Number 19, and it makes 20, which

which I ſubtract from 24, and there remains 4. So is the Difference 4 £. $\frac{9}{10}$, as under.

			EXAMP. 7.	
	£.			£.
From	24 $\frac{5}{10}$		From	419 $\frac{9}{46}$
Take	19 $\frac{6}{10}$		Take	147 $\frac{27}{46}$
Anſ.	4 $\frac{9}{10}$		*Anſ.*	271 $\frac{28}{46}$
Prov'd as in Ex. 1.	24 $\frac{5}{10}$		*Proof*	419 $\frac{9}{46}$.

Phi. *A Proof of* EXAMPLE 6. *by common* Subtraction.

	£.		£. s.	
From	24 $\frac{5}{10}$,	that is,	24 10,	by Caſe 9. of *Reduction.*
Take	19 $\frac{6}{10}$,	that is,	19 12	
Anſ.	4 $\frac{9}{10}$,	that is,	4 18	*Difference.*
Proof	24 $\frac{5}{10}$,	that is,	24 10	

Tyr. This is pretty to ſee the Proof of one Rule of Arithmetic by another.

Phi. You may prove any of the four Rules in *Fractions* by common Arithmetic as well as *this*; for they will come exactly alike if you proceed in a right Manner. Well, *Tyrunculus*, you ſeem to me to be qualified for *Multiplication*, but I have a Fancy to try you with one Queſtion more, which will make you Maſter of *Subtraction*.

EXAMPLE 8.

A *lent* B 240 £. $\frac{2}{5}$ *of* $\frac{3}{7}$ *of* $\frac{5}{9}$; *and* B *paid him* 176 £. $\frac{5}{7}$ *of* $\frac{11}{12}$; *what is ſtill due to* A?

Tyr. I proceed thus:

A 240 £. $\frac{2}{5}$ of $\frac{3}{7}$ of $\frac{5}{9}$; and *B* 176 £. $\frac{5}{7}$ of $\frac{11}{12}$. Now $\frac{2}{5}$ of $\frac{3}{7}$ of $\frac{5}{9} = \frac{30}{315}$; and $\frac{5}{7}$ of $\frac{11}{12} = \frac{55}{84}$. Then $\frac{30}{315}$ and $\frac{55}{84}$, reduced to a common Denominator, will be $\frac{2520}{26460}$ and $\frac{17325}{26460}$. Thus are the *Fractions* prepared, and will ſtand thus:

	£.
A lent *B*	240 $\frac{2520}{26460}$
B paid him	176 $\frac{17325}{26460}$. Then by *Ex.* 6.

I find the Balance 63 $\frac{11655}{26460}$; which, by *Caſe* 9. in *Reduction*, is 63 £. 8 *s.* 9 *d.* $\frac{1}{2}$ $\frac{2263}{2646}$.

Phi. I muſt needs ſay it is a Pleaſure to me to inſtruct you, *Tyrunculus.* I have but one Thing more to obſerve to you, and then we will go to *Multiplication.*

Note, When you are to add or ſubtract the *Fraction* of a Farthing or a Penny from the *Fraction* of a £. or Guinea, &c. then (by *Caſe* the 8th. in *Reduction*) reduce the *Fraction* of the one, and make it equivalent to the *Fraction* of the other, and add or ſubtract as the Queſtion requires, you have the Anſwer. And thus much for *Subtraction.*

SECT. III.

MULTIPLICATION *of* VULGAR FRACTIONS.

Tyr. HOW is *Multiplication* of *Fractions* performed?

Phi. As in common Arithmetic, ſo alſo here are two Parts or Factors given, *viz.* the *Multiplicand* and *Multiplier.*

Multiplier. When therefore you have reduced the mixt Numbers to *improper Fractions*, and the *compound* to *ſimple Fractions*, the Rule is, multiply the two Numerators together for a *new Numerator*, and the Denominators together for a *new Denominator*, and you have the Product or Anſwer; which, if it be an *improper Fraction*, reduce to a mixt Number, and the Work is done.

EXAMPLE I.

Multiply $\frac{4}{5}$ *by* $\frac{5}{8}$.

Anſ. $\frac{20}{40} = \frac{1}{2}$.

EXAMPLE 2.

Multiply $\frac{42}{56}$ *by* $\frac{24}{38}$.

Here $42 \times 24 = 1008$ N. N. and $56 \times 38 = 2128$ N. D. *Anſ.* $\frac{1008}{2128}$.

Tyr. Nothing is eaſier than this indeed. Pray try me with a few Queſtions?

Phi. I will; and I am fully perſuaded that you will work moſt of them, if you rightly obſerve the Rule, which pray look at once more, leſt you be not perfect in it.

EXAMPLE 3.

Multiply $\frac{4765}{9}$ *by* $\frac{14}{29}$.

Tyr. Firſt $4765 \times 14 = 66710$ for N. N. and $9 \times 29 = 261$ N. D. *Anſ.* $\frac{66710}{261} = 255\frac{155}{261}$.

Phi. You are very right.

EXAMPLE 4.

Multiply $\frac{5}{4}$ *of* $\frac{8}{5}$ *by* $\frac{3}{6}$ *of* $\frac{5}{9}$.

Tyr. Firſt $\frac{5}{4}$ of $\frac{8}{5} = \frac{40}{20}$; and $\frac{3}{6}$ of $\frac{5}{9} = \frac{15}{54}$; therefore I multiply $\frac{40}{20}$ by $\frac{15}{54}$, and the *Anſwer is* $\frac{600}{1080}$. Now I'll aſk you one, if you pleaſe.

Phi. Pray do.

Ex-

EXAMPLE 5.

Tyr. *Multiply* 41 £. $\frac{4}{5}$ *by* 12 £. $\frac{3}{4}$.

Phi. Thefe being mixt Numbers, they are equal to *improper Fractions*, that is, 41 $\frac{4}{5} = \frac{209}{5}$; and 12 $\frac{3}{4} = \frac{51}{4}$; that is, $\frac{209}{5}$ and $\frac{51}{4}$. Now $209 \times 51 = 10659$; and $5 \times 4 = 20$; fo that the *Anfwer* is $\frac{10659}{20}$, which reduced to a mixt Number, is 532 $\frac{19}{20}$; that is 532 £. 19*s*.

Tyr. Now I think I can do any Queftion in this Rule.

Phi. Perhaps fo; but it runs in my Head that I can puzzle you in one of the two next Examples.

EXAMPLE 6.

Multiyly 14 £. $\frac{3}{4}$ *of* $\frac{5}{6}$, *by* 9 £. $\frac{3}{5}$ *of* $\frac{15}{18}$.

Tyr. I know I can do this. Firft, $\frac{3}{4}$ of $\frac{5}{6} = \frac{15}{24}$, *viz.* $= \frac{5}{8}$; and $\frac{3}{5}$ of $\frac{15}{18} = \frac{45}{90}$, *viz.* $= \frac{1}{2}$. Therefore I multiply 14 £. $\frac{5}{8}$ by 9 £. $\frac{1}{2}$. Thefe reduced to *improper Fractions* will ftand thus; multiply $\frac{117}{8}$ by $\frac{19}{2}$. Now $117 \times 19 = 2223$ N. N. and $8 \times 2 = 16$ N. D. that is, $\frac{2223}{16}$ *Anf.* $= 138 \frac{15}{16} =$ 138 £. 18 *s.* 9 *d.*

Phi. Very well done; and the better becaufe you abbreviated the *Fractions* $\frac{45}{90}$ and $\frac{15}{24}$; for which you will fee the Reafon given in the Method of *Abbreviations*, *Dialogue* 4. *Sect.* 2. Now for the fecond Queftion, *Tyrunculus.*

EXAMPLE 7.

Multiply 14 £. $\frac{5}{9}$ *by* 13 £.

Tyr. Let me fee 14 £. $\frac{5}{9}$ by 13 £.—Why here is but one *Fraction given.*—I believe you have fet me now indeed.

Phi.

Phi. It was but this Inſtant that you ſaid, you believ'd you could do any Sum in the Rule.—Why don't you make a *Fraction* of the whole Number 13?

Tyr. I muſt know how firſt.

Phi. O for Shame, don't you know, ſo plainly as I told you in *Dial* 2. *Obſerv.* 4. Pray don't forget your former Inſtructions, and then blame me; I thought this would put you to a Nonplus: However, it is the firſt great Error you have committed, ſo we will paſs it by; but pray be more careful for the Future. I told you to make a *Fraction* of a whole Number, is only putting Unity under it. Thus 13 is $= \frac{13}{1}$.

Tyr. I bluſh to think I ſhould be ſo remiſs. However, pardon me, I know now eaſily how to perform it: For $14\frac{5}{9} = \frac{131}{9}$; and $13 = \frac{13}{1}$; therefore by multiplying $\frac{131}{9}$ by $\frac{13}{1}$, I have $\frac{1703}{9} = 189\frac{2}{9}$; that is, 189 £. 4*s.* 5*d.* $\frac{1}{4}\,\frac{3}{9}$ or $\frac{1}{3}$ of a Farthing *Anſ.*

Phi. Very right, *Tyrunculus*; and now I ſhall make ſome Obſervations, which, tho' already known to ſuch as are well verſed in *Vulgar Fractions*, yet, as they are not taken Notice of by any Author I am acquainted with, it may be ſerviceable to you and others.

NOTE I.

When one *ſimple Fraction* is multiplied by another, the Product will be a *ſimple Fraction*, therefore the Anſwer is leſs than Unity. It will be alſo leſs than Unity when one *compound Fraction* is multiplied by another, provided they be compounded of *ſimple Fractions*. (See *Obſerv.* 1. *Dial.* 2.) Thus in *Example* 1 where $\frac{4}{5}$ is multiplied by $\frac{5}{8}$, the Anſwer is but $\frac{1}{2}$; that is, $\frac{4}{5}$ of a £. multiplied by $\frac{5}{8}$ of a £. is but $\frac{1}{2}$ of a £. or 10 Shillings.

NOTE 2.

Contrary to this, one *improper Fraction* multipled by another is more than Unity. Thus if you take the *Fractions* in *Example* 1. and invert them, it will be $\frac{5}{4}$ of a £. multiplied by $\frac{8}{5}$ of a £. and the Anſwer will be $\frac{40}{20} = 2$ whole Integers, or 2 £. So that the Product now is juſt four Times more than it was before.

NOTE 3.

When a *ſimple Fraction* is to be multiplied by or with an *improper Fraction*, the Product will be ſometime a *ſimple*, and ſometimes an *improper Fraction*. Thus $\frac{4}{9}$ multiplied by $\frac{11}{8}$ produces the *ſimple Fraction* $\frac{44}{72}$; but $\frac{9}{7}$ multiplied by $\frac{5}{6}$ gives $\frac{45}{42}$ an *improper Fraction* for the Anſwer.

NOTE 4.

When the Numerator of one *Fraction* is equal to the Denominator of the other, and the Denominator of it equal to the Numerator of the other, then will their Product be Unity or 1. Thus $\frac{4}{9}$ multiply'd by $\frac{9}{4}$, the Product is $\frac{36}{36} = 1$. (See *Dial* 2. *Obſerv.* 3.) So alſo $\frac{7}{11}$ of a £. into $\frac{11}{7}$ of a £. $= \frac{77}{77} = 1$ £.

Tyr. Theſe Obſervations are of great Service to ground any one in the right Notion of *Multiplication*. But pray is there no Method to prove this Rule?

Phi. Certainly there is; and this is the Beauty of *Arithmetic*, that it admits of the Proof of itſelf divers Ways by diverſe Rules. For Inſtance, Suppoſe $\frac{28}{7}$ of a £. were multiplied by $\frac{15}{3}$ of a £. the Anſwer is $\frac{420}{21} = 20$ £.

PROOF.

PROOF.

Now $\frac{28}{7} = 4$ £. and $\frac{15}{3} = 5$, and $4 \times 5 = 20$ £. as above, &c. &c. &c.

Tyr. I am highly oblig'd to you *Philomathes*, for your Care in giving me ſo many Examples. Pray is there any Thing more worthy my Notice, or neceſſary to be known in *Multiplication?*

Phi. Nothing: I have indeed been more particular already than I intended; therefore I ſhall paſs directly to *Diviſion.*

SECT. IV.

Of DIVISION *of* VULGAR FRACTIONS.

Tyr. HOW do you divide one *Fraction* by another?

RULE I.

Phi. After having reduced all *mixt Numbers* and *compound Fractions* as before directed, to *ſimple* or *improper Fractions*, the Rule is, Multiply the Numerator of the *Fraction* to be divided into the Denominator of the *Fraction* you divide by, and place their Product for a new Numerator; then multiply the Denominator of the Dividend into the Numerator of the Diviſor for a new Denominator, which place under the new Numerator for an Anſwer. *Or,*

RULE

RULE 2.

If you invert the Divifor, that is, turn it into contrary Order, by fetting the Numerator underneath, and the Denominator over it; then multiply the Numerators and Denominators together, as in *Multiplication*, and you have the fame Anfwer as above.

EXAMPLE. 1. *by* RULE 1.

Divide $\frac{4}{9}$ *by* $\frac{7}{11}$. *Anf.* $\frac{44}{63}$.

Same EXAMPLE *by* RULE 2.

Multiply $\frac{4}{9}$ *by* $\frac{11}{7}$. *Anf.* $\frac{44}{63}$.

EXAMPLE 2. *by* RULE 1.

Divide $\frac{14}{9}$ *by* $\frac{6}{11}$. *Anf.* $= 2\frac{46}{54}$.

Same EXAMPLE *by* RULE 2.

Multiply $\frac{14}{9}$ *by* $\frac{11}{6}$. *Anf.* $\frac{154}{54}$ *as before.*

Tyr. I like the fecond as well as the firft Way.

Phi. Ufe which you pleafe, providing you are but perfect in either. There is no Occafion for any more Examples, feeing that you have a Rule both for *fimple* and *compound Fractions.* However, I'll give you an Example or two more by Way of Exercife.

Ex-

EXAMPLE 3.

Divide 41 £. $\frac{3}{5}$ *by* 6 £. $\frac{5}{8}$; that is, *divide* $\frac{208}{5}$ *by* $\frac{53}{8}$.

Anſ. $\frac{1664}{265} = 6$ £. $\frac{74}{265}$; that is, 6 £. 5 *s.* 7 *d.* $\frac{5}{265}$.
And after the ſame Manner for *compound Fractions.*

Tyr. I underſtand you quite well. Pray is there any Thing to be obſerv'd in *Diviſion?*

Phi. I ſhall make a few Remarks upon the Rule itſelf, which may be of Service.

NOTE 1.

When an *improper Fraction*, having Unity for its Denominator, is to be divided by a *ſimple Fraction*, whoſe Numerator is alſo Unity, the Quotient will always be an *improper Fraction*, having Unity for its Denominator, and therefore conſequently equal to a *whole Number.*

NOTE 2.

When the Denominators or Numerators are not Unity, the Quotient will ſometimes be an *improper*, and ſometimes a *ſimple Fraction.*

NOTE 3.

From hence it is eaſy to perceive, that *Diviſion* of *Fractions* will anſwer the ſame End as common *Multiplication:* That is, a leſs Number may be brought into greater by this *Diviſion*, contrary to

common

common *Division, viz.* Moidores, Guineas, or Pounds Sterling into Pence and Farthings; or Hundred-Weight into Pounds and Ounces, *&c.*

Tyr. What do you say, Pounds may be brought into Pence and Farthings by *Division?* I thought *Division* had made any Number *less*, and not *more!*

Phi. It is true, it does so in common *Division*, but it is quite contrary in *Vulgar Fractions*; for *here*, *more* is brought into *less* by *Multiplication*, and *less* into *more* by *Division*.

Tyr. I should be glad to see an Example of this Sort if you please; for I have heard some great Pretenders to *Arithmetic* say it cannot be done, 'tis contrary to Reason.

Phi. Please then to propose a Question yourself?

EXAMPLE I.

Tyr. *It is required to bring* 30 *Moidores into Farthings by* Division *only?*

Phi. And cannot you do it think you?

Tyr. —— Why really at present I am at a Loss.

Phi. Pray be pleased to read over *Note* 1. in *Multiplication* and *Division*, for it is only for Want of being perfect in them, and truly understanding the Nature of different *Fractions*.

Observe then,

As one Part of the given Question is Farthings, and the other Moidores, I reduce (by *Case* 8 in *Reduction*) a Farthing to the *Fraction* of a Moidore, which *Fraction* I make a Divisor; and the whole Number

Number 30 I make a *Fraction* of alſo for a Dividend; ſo will the Quotient be the true Anſwer. *See the Work.*

By *Caſe* 6. of *Reduction* $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{27} = \frac{1}{1296}$ of a Moidore for a Diviſor; and 30 will be $\frac{30}{1}$. Now $\frac{30}{1} \div \frac{1}{1296} = \frac{38880}{1}$; that is, 38880 Farthings. Do you underſtand it?

Tyr. Yes, quite well; I could not have thought it had been ſo eaſy: But pray what does this Mark $\div$ ſignify?

What $\div$ ſignifies.

Phi. It is the Sign of *Diviſion*, and ſhews, that the Number before it is to be divided by the Number after it.

Tyr. Very Well Pray try me with a Queſtion of this Sort?

EXAMPLE 5.

Phi. *It is required to bring* 84 *Guineas into Farthings by* Diviſion *only.*

Tyr. I find (by *Caſe* 6. of *Reduction*) a Farthing to be equal to $\frac{1}{1008}$ of a Guinea. Then $\frac{84}{1} \div \frac{1}{1008} = 84672$ Farthings *Anſ.*

Phi. Very right, and pray remember that the ſame is to be done by *Decimal Fractions*, by finding the Decimal of a Farthing at a Moidore or Guinea the Integer, and dividing the *whole Number* thereby you will have the ſame Anſwer.

From hence will naturally ariſe the ſelf-evident Truth of

NOTE 4.

That when any *whole Number* is divided by a *ſimple Fraction*, the Quotient will be ſo much larger than the Dividend as the Diviſor is leſs than Unity; but when a *ſimple Fraction* is divided by a *whole Number*, the Quotient will be ſo many Times leſs than the Dividend as the Diviſor exceeds Unity. Thus 5 divided by $\frac{1}{4}$ is equal to 20; but $\frac{1}{4} \div 5 =$ but $\frac{1}{20}$.

And thus, *Tyrunculus*, having finiſhed theſe four Rules with Variety of Examples, I ſhall now exerciſe you in them with ſome practical Queſtions in the *Rule of Proportion*; which, if duly obſerved, will make you a compleat Maſter of *Vulgar Fractions.*

DIA-

DIALOGUE IV.

SECT. I.

The RULE *of* 3, *in* VULGAR FRACTIONS.

Tyr. I Am proud to think I am got thus far, and yet I almoſt dread the Queſtions you are going to ſet me.

Phi. If you be perfect in the foregoing Rules, you have no *Reaſon* to fear *this* at all; for it is nothing elſe but putting the others in Practice.

Tyr. Is not the Rule of 3. of *Fractions* wrought in the ſame Manner as the common Rule of 3. direct?

Phi. The very ſame, due Regard being had to the *Fractions.* There are two Methods, the ſecond of which is (in general) the readieſt and eaſieſt; but you may take your Choice.

RULE 1.

Having reduced all *compound* to *ſimple Fractions*, and all *mixt Numbers* to *improper Fractions*, then ſtate your Queſtion by making the firſt and third Number of one Name or Denomination; this done, *Multiply your ſecond Number by your third, and divide by your firſt*, and you have the Anſwer. *Or*,

RULE 2.

Having reduced the *Fractions*, and placed the Numbers in Order as before directed, *Multiply the Denominator of your firſt Number into the Numerators*

of the second and third for a new Numerator; then multiply the Numerator of the first Fraction or Number into the Denominator of the second and third, for a new Denominator, which place under the new Numerator for an Answer.

EXAMPLE I.

If $\frac{3}{4}$ *of a Yard cost* $\frac{5}{6}$ *of a £. what cost* 25 $\frac{5}{8}$ *Yards.*

$$\text{If } \tfrac{3}{4} \text{ —— } \tfrac{5}{6} \text{ —— } 25\tfrac{5}{8}.$$

$$\begin{array}{r} 8 \\ \hline 205 \\ \hline 8 \end{array}$$

Now $\frac{205}{8} \times \frac{5}{6} = \frac{1025}{48}$; this $\div \frac{3}{4} = \frac{4100}{144} =$ 28 £. $\frac{68}{144}$ = 28 £. 9 *s.* 5 *d.* 1 *q.* $\frac{48}{144} = \frac{1}{3}$.

Or, by RULE 2.

Having stated the Question thus, If $\frac{3}{4} - \frac{5}{6} - \frac{205}{8}$. *I multiply the Denominator of my first Number* (*viz* 4.) *into the Numerators of the second and third* (*viz.* 5 *and* 205) *and it gives* 4100 *for a N. Numerator. Then I multiply the Numerator of the first* (*viz.* 3) *into the Denominators of the second and third,* (*viz.* 6 *and* 8) *and it gives* 144 *for a N. Denominator. So is the Answer* $\frac{4100}{144}$, = 28 £. $\frac{68}{144}$ *as above.*

Tyr. I think as you say this second Method is the best, if it were only because it saves the Trouble of *Division.*

Phi. It is the best Way in your plain easy Questions, but in some Respects the first Method is most practicable; however, either Way you see answers the

the ſame End, and therefore you may take either of them, as Practice or Fancy may direct.

Tyr. I ſhall take Care to be perfect in both. Pleaſe to try me with a Queſtion?

Phi. I will.

EXAMPLE 2.

If a Load of Wheat coſt 7 £. $\frac{14}{24}$, *what coſt one Buſhel?*

Tyr. A Load being 40 Buſhels, I ſtate it thus:

If $\frac{40}{1}$ —— £. 7 $\frac{14}{24}$ —— $\frac{1}{1}$

24

182

24 Now $\frac{182}{24} \times \frac{1}{1} = \frac{182}{24}$; this $\div \frac{40}{1} = \frac{182}{960}$ of a £. which reduced, the Value of it is 3 *s.* 9 *d.* $\frac{1}{2}$ *Anſ.*

SECOND WAY.

Firſt I reduce the *mixt Number* 7 $\frac{14}{24}$ to an *improper Fraction*, and it is $\frac{182}{24}$, as above, then the Numbers will ſtand thus:

If $\frac{40}{1}$ —— $\frac{182}{24}$ —— $\frac{1}{1}$.

Now $1 \times 182 \times 1 = 182$ N. N. and $40 \times 24 \times 1 = 960$ N. D. So is the *Anſwer* $\frac{182}{960} = 3$ *s.* 9 *d.* $\frac{1}{2}$, as before.

Phi. Very well done, *Tyrunculus*?

Ex-

EXAMPLE 3.

What is the Intereſt of 219 £. $\frac{3}{5}$ *for a Year at* 5 £. $\frac{3}{8}$ *per Cent.*

Tyr. —— Here I muſt crave your Aſſiſtance.

Phi. You ſhall have it in Words at length.

Firſt, If $\frac{100}{1}$ —— 5 $\frac{3}{8}$ —— 219 $\frac{3}{5}$. My 2*d.* and 3*d.* Terms being *mixt Numbers*, I reduce them to *improper Fractions*, and they are $\frac{43}{8}$ and $\frac{1098}{5}$. Now $\frac{43}{8} \times \frac{1098}{5} = \frac{47214}{40}$; this $\div$ the 1*ſt.* Number $\frac{100}{1}$ gives $\frac{47214}{4000}$, which reduced to a *mixt Number* is 11 £. $\frac{3214}{4000}$, *viz.* 11 £. 16 *s.* 0 *d.* 3 *qrs.* $\frac{1440}{4000} = \frac{144}{400} = \frac{9}{25}$. Try it at Leiſure by the ſecond Method.

Tyr. I could not have thought *Vulgar Fractions* had been ſo uſeful.

Phi. Nothing more neceſſary than theſe and *Decimal Fractions*, for the ready finding the Intereſt or Value of any Thing, eſpecially when the Queſtions are not in whole Numbers, as you will ſee by the following Examples.

EXAMPLE 4.

A Merchant makes an Aſſurance upon a Ship and Cargo (bound to a certain Port) valued at 4500 £. 15 *s. and agrees to pay* 16 *Guineas* per Cent. *what comes the Premium or Charge of the Aſſurance to?*

Firſt, 16 Guineas being 16 £. 16 *s.* this in *Fraction* is 16 $\frac{16}{20} = 16\ \frac{4}{5}$; and 4500 £. 15 *s.* is 4500 $\frac{3}{4}$. This and 16 $\frac{4}{5}$ reduced to *improper Fractions* will be $\frac{84}{5}$ and $\frac{18003}{4}$. Then will the Number ſtand thus:

If $\frac{100}{1}$ —— $\frac{84}{5}$ —— $\frac{18003}{4}$.

Now

Now by *Rule* 2. $1 \times 84 \times 18003 = 1512252$ N. N. And $100 \times 5 \times 4 = 2000$ N. D. So is $\frac{1512252}{2000}$ the Anſwer, which reduc'd to a *mixt Number*, you have £. 756 $\frac{252}{2000}$ = 756 £. 2 *s.* 6 *d.* $\frac{480}{2000}$ or $\frac{48}{200}$.

EXAMPLE 5.

A *buys of* B £. 420 $\frac{2}{3}$ *Stock, and gives* £. 95 $\frac{4}{5}$ per Cent. *what comes it to?*

Firſt, 420 $\frac{2}{3} = \frac{1262}{3}$, and 95 $\frac{4}{5} = \frac{479}{5}$. Then,

If $\frac{100}{1}$ —— $\frac{1262}{3}$ —— $\frac{479}{5}$.

Anſ. $\frac{604498}{1500}$ = 402 £. 19 *s.* 11 *d.* 2 *qrs.* $\frac{108}{150}$.

The Proof of this is worthy your Obſervation, *Tyrunculus*, to ſhew you the Beauty of *Fractions*, which I inſert purely for your Satisfaction, to give you a juſt Idea of Things of this Nature.

PROV'D *another* WAY.

Firſt, £. 95 $\frac{4}{5}$ *per Cent.* wants £. 4 $\frac{1}{5}$ *per Cent.* of being *Cent. per Cent,* therefore the Queſtion may be read thus:

What come £. 420 $\frac{2}{3}$ *to, deducting* £. 4 $\frac{1}{5}$ per Cent. Then,

If $\frac{100}{1}$ —— 4 $\frac{1}{5}$ —— 420 $\frac{2}{3}$.

Work as in the laſt, and you will have $\frac{26502}{1500}$ = 17 £. 13 *s.* 4 *d.* 1 *q.* $\frac{420}{1500}$ *Anſwer.* Now this added to the foregoing Sum 402 £. 19 *s.* 11 *d.* 2 *qrs.* $\frac{108}{150}$ is equal to the original Stock propoſed, *viz.* £. 420 $\frac{2}{3}$ = 420 £. 13 *s.* 4 *d.*

Tyr. Indeed, *Philomathes*, this is delightful; pray continue your Examples. *Phi.*

Phi. I am as ready to do it as you are to aſk; but remember *Tyrunculus*, (by Way of Digreſſion) *We muſt cut our Garment according to the Cloth*: I have already given more Examples than I intended; but ſtill, if you have any particular Queſtion to aſk me, I am ready to do any Thing that may be of Service.

Tyr. Sir, you are extremely kind to indulge me thus far; but what I have to add is this, That by what I have ſeen of this Rule, it muſt be very ſerviceable to tell the Nature or Proportion of Coins: Is it not?

Phi. To be ſure it is, eſpecially when the different Sorts of Exchange with ſome other current Money is not altogether equal to our Pound *Sterling*.

Tyr. I will aſk you a Queſtion then.

EXAMPLE 6.

A Merchant in Holland *draws a Bill upon his Correſpondent in* London *for* 4280 *Ductoons, at* 6 *s.* 3 *d.* $\frac{3}{5}$ *each; what muſt he receive in Pounds* Sterling?

If $\frac{1}{1}$ ——— 6 *s.* 3 *d.* $\frac{3}{5}$ ——— $\frac{4280}{1}$.

Firſt, bring the ſecond Number into Pence, then multiply them by the Denominator 5, and take in the Numerator 3, ſo will the Numbers be $\frac{1}{1}$ ——— $\frac{378}{5}$ and $\frac{4280}{1}$. Now, by *Rule* 2. $1 \times 378 \times 4280 = 1617840$ N. N. And $1 \times 5 \times 1 = 5$ N. D. So is $\frac{1617840}{5}$ the Anſwer in Pence, *viz.* 323568 = 1348 £. 4 *s.* 0 *d.*

Tyr. Pray prove the Work by whole Numbers?

Phi. That is done very eaſily by *Practice*, or ſeveral other Ways. But for common Underſtanding, I know none better than this: Firſt, find the Value

Value of 4280 Pieces, at 6 *s.* 3 *d.* (that is, at 75 *d.*) each, and it is 321000 *d.* Then you have got to find the Value of 4280 Pieces, at $\frac{3}{5}$ of a Penny each; therefore by multiplying 4280 by 3, and dividing by 5, you have 2568 Pence, which added to the other, will give you the Anſwer as follows:

4280 Pieces	at 75 *d.*	=	321000
Ditto,	at $\frac{3}{5}$ *d.*	=	2568
Ditto,	at 75 $\frac{3}{5}$ *d.*	=	323568 *d.* = 1348 £. 4 *s.* 0 *d.* as above.

Tyr. Then I am always to multiply by the Numerator and divide by Denominator in ſuch Caſes; am I not?

Phi. I know no eaſier or ſhorter Way I aſſure you.

Tyr. It is eaſy enough indeed as you ſay, and I am oblig'd to you for ſo plain a Demonſtration: Give me Leave to aſk you a Queſtion ſtarted the other Day in my Company, and I'll have done. It is this:

EXAMPLE 7. *

A poor Man dying leaves 20 *Shillings to his four Sons,* A, B, C, *and* D; *to* A *he left* $\frac{1}{3}$, *to* B $\frac{1}{4}$, *to* C $\frac{1}{5}$, *and to* D $\frac{1}{6}$, *with a particular Charge that the Whole might faithfully be diſtributed among them; it is demanded what each Legacy amounts to?*

Phi. I have not room to inſert the whole Work, but will plainly tell you the Method of doing any Thing of this Sort.

Firſt,

* This and the foregoing Example were added by Deſire of a Friend.

First, I take the $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ of 20 *s.* and adding the Parts together, I find the Sum but 19 *s.* So that the *Executor* has 1 *s.* in Hand, and the Will is not fulfilled. Then ſay, If 19 *s.* give $\frac{1}{3}$, (*viz.* 6 *s.* 8 *d.*) what will 20 *s.* give? Proceed thus with the reſt, and you will find the Anſwer to be as under:

A's Share	0	7	0	$\frac{4}{19}$
B's — —	0	5	3	$\frac{3}{19}$
C's — —	0	4	2	$\frac{10}{19}$
D's — —	0	3	6	$\frac{2}{19}$
Sum	1	0	0	$\frac{0}{0}$ *Anſ.*

This ſhews the Parts are not always equal to the Whole, but ſometimes leſs and ſometimes more; for had he left the $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and the $\frac{1}{5}$ of 20 *s.* it would amount to 25 *s.* 8 *d.* which you ſee is 5 *s.* 8 *d.* more than the Whole, though each Part is ſeparately ſuch an equal Part of the Whole without any Remainder. Their Shares then of this 5 *s.* 8 *d.* are found as above, only now ſaying, If 25 *s.* 8 *d.* be 20 *s.* what will $\frac{1}{2}$ or 10 *s.* be, &c.

Note, By this ſame Method is found the particular Shares of every Creditor when a Man breaks or becomes a Bankrupt; as alſo how much in the £. his Effects amount to.

Tyr. I return you hearty Thanks, Sir, for your Care; and do aſſure you that I underſtand the Nature of what I have ſeen very well.

Phi. I am glad of it, *Tyrunculus*; but you will be more confirm'd in the Knowledge of *Fractions*, and will know better how to apply them to Uſe, if I ſhew you ſomething of the Nature of *Abbreviations.*

SECT.

SECT. II.

Of ABBREVIATIONS.

Tyr. WHAT is the Uſe of *Abbreviation?*

Phi. The right Underſtanding of *Abbreviations* will mightily help to ſhorten the Work, and ſave a Multiplicity of Figures; beſides, without them it would be quite difficult to perform ſome Queſtions, at leaſt very tedious, as you'll ſee by and by.

Tyr. What would you have me abbreviate the *Fractions* before I begin the Queſtion?

Phi. To be ſure I would when they will eaſily allow of it; for if you remember you did it yourſelf in *Example* 6. in *Multiplication.*

Tyr. I believe I did; however, let me have an Example or two, that I may the better underſtand it.

Phi. You ſhall.

EXAMPLE I.

If $\frac{84}{140}$ *of a Load coſt* $\frac{70}{175}$ *of a £. what coſt* 12 $\frac{36}{96}$

Firſt, $\frac{84}{140} = \frac{3}{5}$, $\frac{70}{175} = \frac{2}{5}$, and $\frac{36}{96} = \frac{3}{8}$. Therefore the Sum may be thus read:

If $\frac{3}{5}$ *of a Load coſt* $\frac{2}{5}$ *of a £. what coſt* 12 $\frac{3}{8}$ *Loads?*

Anſ. £. 8. $\frac{3}{12}$ = 8 £. 5 *s.* 0 *d.* Now is it not eaſier to work with $\frac{3}{5}$, $\frac{2}{5}$, and $\frac{3}{8}$, than with the original *Fractions* $\frac{84}{140}$, $\frac{70}{175}$, and $\frac{36}{96}$?

Tyr. To be ſure, the Thing appears plain; and I ſhall take Care to obſerve it. Is this all you have to ſay upon *Abbreviations?*

Phi. No; I have ſomething more to add, wherein you will ſee the Beauty and Uſe of *Abbreviations* yet more plainly: Beſides, your Friend *Diſcipulus* deſir'd me to communicate it to you, as very few or none have taken Notice of it in their Writings.

Tyr. Pray what is it?

Phi. It is, *To know whether a* Fraction, *when abbreviated, (or reduced to its loweſt Terms) be equivalent in all Reſpects to the original, or given* Fraction.

Tyr. I know no Method of proving it, but by comparing the Value of one with the Value of the other; thus I find $\frac{24}{36}$ of a £. is equal to 13 *s.* 4 *d.* and I find $\frac{2}{3}$ of a £. to be the ſame; therefore I conclude that $\frac{2}{3}$ is $= \frac{24}{36}$.

Phi. It is right; but ſuppoſe you could neither abbreviate a *Fraction*, nor yet find its Value, how then would you act?

Tyr. That I do not know.

Phi. Indeed, *Tyrunculus*, though you can work the Rule of Proportion pretty well, you are no great Judge of the Nature of it I find. Obſerve then,

Firſt, *As the Numerator of the* Fraction, *in its loweſt Terms, is to its Denominator, ſo will the Numerator of the original or given* Fraction *be to its own Denominator: Or, as one Numerater to the other, ſo will one Denominator be to the other,* &c.

Ex-

EXAMPLE 2.

To prove whether $\frac{3}{5}$ *be equal to* $\frac{84}{140}$, *or* = $\frac{127}{180}$ $\frac{30480}{43200}$.

First, as 3 to 5, so is 84 to 140: Or, as 3 to 84, so is 5 to 140, *&c.* Again, As 127 to 180, so is 30480 to 43200, *&c. &c. &c.*

Tyr. I am yet more oblig'd to you *Philomathes*; for this must infallibly prove what you have said sure enough.

Phi. Since you are sensible of this, *Tyrunculus*, I will shew you another Way to prove it much shorter and easier than the former.

Second, *When you have reduced any* Fraction *to its lowest Terms to prove whether it be right, multiply the Numerator of the original* Fraction, *by the Denominator of the abbreviated one; and the Denominator of the Original, by the Numerator of the abbreviated one; and if the Products are equal, your Work is rightly performed.* Thus, Take the two *Fractions* above, *viz.* $\frac{84}{140} = \frac{2}{5}$. For $2 \times 140 = 280$ and $5 \times 84 = 280$. Again, $\frac{30480}{43200} = \frac{127}{180}$: For 30480×180, and 43200×127 are both equal to 548640.

Tyr. This is short and easy indeed!

Phi. I will now give you an Example or two to shew you that the Knowledge of *Abbreviations* are of more Use than you imagined.

EXAMPLE 3.

A and B *are two Merchants, but of different Places;* A *owes* B 46 £. 12 *s.* 9 *d.* *Now* £. 100 *of* A's *current Money is equal to* 140 £. *of* B's; *what must* A *pay to remit the aforesaid Debt?*

See the Work.

	£.	s.	d.
	46	12	9
			5
7)	233	3	9
	33	6	3 *Ans.*

Tyr. This is short indeed!

Phi. But you know the Reason (I hope) of my multiplying and dividing by these Figures; do you not?

Tyr. Stay, let me consider a little upon it.——I see the Reason now plain enough. They are in Proportion to each other as the current Money of the Places are, I perceive; that is, $\frac{100}{140} = \frac{5}{7}$; for as 5 to 7, so is 100 to 140. Is not this the Reason?

Phi. Be sure it is; for I can multiply and divide better by 5 and 7, than by 100 and 140; besides, how much shorter is this, than to work it at Length by the *Rule of* 3. direct. So also if 108 £. of *A*'s be equal to 45 £. of *B*'s, then I multiply by 5, and divide by 12, because $\frac{45}{108} = \frac{5}{12}$, and the Answer will be 19 £. 8 *s.* 7 *d.* $\frac{3}{4}$. Try you at it, supposing 105 £. of *A*'s be equal to 165 £. of *B*'s.

Tyr. I will, Sir, and I heartily thank you for this additional and useful Observation.

Phi. The Pains I have taken I shall count a Pleasure, if you make but a good Improvement: And

let

let me perſuade you not to meddle with *Algebra* till you are perfect in *Fractions*; for if you do, you will not be able to make a right Judgment of the *Problems*, much leſs know how to do them. If you think you underſtand what you have done, there remains but one Thing more before you enter upon *Algebra*, and that is, that you learn the *Signs* and *Characters* therein uſed.

Tyr. You have ſhewn me ſome already.

Phi. I know it: But there are a great many more; therefore I ſhall write them down, and give you them Home, that you may learn them by Heart before I ſee you again; which, though I am always glad of, yet, upon this Occaſion, I neither deſire nor expect, till you have learnt them ſo perfectly, as to know their Meaning the Moment you ſee them.

Tyr. You may depend upon it, *Philomathes*, in a ſhort Time: Where be they?

Phi. Stay a little.———Here, *Tyrunculus*, here they are, and I wiſh you well to learn them.

Tyr. I heartily thank you, *Philomathes*, and am your Servant.

Phi. I am yours, *Tyrunculus*.

SECT. III.

An EXPLANATION *of the* PRINCIPAL SIGNS *and* CHARACTERS *uſed in* ALGEBRA.

1. THIS Character (+) is the Sign of *Addition*, and ſignifies, that the Numbers or Quantities between which it is placed, are to be added together in one Sum. Thus, 3 + 5 ſhews, that 3 and 5 are to be added together. It ſtands for the Word *more* alſo. Thus, 5 + 4 + 7, is read, 5 more

more 4, more 7, which make 16: So alſo, $a + b + c + d$, ſhews, that a, b, c, and d, are to be added together.

2. This Character ($-$) is the Sign of *Subtraction*, and ſignifies, that the Numbers or Quantities which come after it, are to be taken from the Numbers or Quantities which ſtand before it. Thus, $a + b - c$, ſhews, that the Quantity c is to be taken from the Sum of a and b. It ſtands for the Word *leſs* alſo. Thus, $9 - 5$, is read, 9 *leſs* 5, which is 4; and $b - 9$ is b *leſs* 9, and ſhews that 9 is to be taken from the Quantity b, &c.

Note farther, That ($+$) ſignifies a *poſitive* or *affirmative* Quantity, or *abſolute* Number; but ($-$) ſignifies a *fictitious* or *negative* Quantity or Number; a Want or Deficiency. Thus -8 is 8 Times leſs than Nothing. So that any Number or Quantity with the Sign $+$ being added to the ſame Number or Quantity with the Sign $-$, their Sum will be equal to Nothing. Thus 8 added to -8 is equal to (0); but -8 taken from $+8$ is $= 16$. (*See Caſe 2d. in Addition and Subtraction of Algebra.*) Again,

An Aſteriſk (*) is frequently uſed for a Cypher in *Subtraction*, that is, $b + c$ taken from $b + c$, there remains * or Nothing.

2. This Character ($\times$) is the Sign of *Multiplication*. It ſignifies *into*, or *multiplied by*. Thus, $4 \times 5 \times 3$, ſhews, that 4 is to be multiplied by or into 5, and their Product into or by 3. So alſo $a \times b \times c \times d$, ſhews the continual *Multiplication* of a, b, c, and d.

☞ When Quantities are placed one after another, without any Sign or Character, it ſhews their *Multiplication*. Thus ab is $a \times b$, or a multiply'd by b.

So

So *abcd* ſhews the Product of *a* into *b* into *c* into *d*, &c. Joining of Quantities therefore is multiplying them together.

4. This Character ($\div$) is the Sign of *Diviſion*, and ſignifies that the Numbers or Quantities before it, are to be divided by the Numbers or Quantities after it. Thus $a \div b$, ſhews that *a* is to be divided by *b*; ſo $16 \div 4$, ſhews that 16 muſt be divided by 4.

Note, There is a better Way of expreſſing *Diviſion*, and it is more frequently uſed; and this is by placing the Dividend a-top, and the Diviſor underneath it. Thus *a* divided by *b* is ſet thus, $\frac{a}{b}$. So alſo 16 divided by 4 is thus placed, $\frac{16}{4}$, &c. &c.

5. Theſe two Lines ($=$) are the Signs of Equality, and ſignify, that the Quantities and Numbers on the one Side of it are equal to the Numbers or Quantities on the other.

6. This Character ($\div\!\div$) is the Sign of *continued* or *Geometrical Proportion*. Thus, $a \div\!\div b \div\!\div c$, &c. are Quantities in *Geometrical Proportion*. But *Geometrical Proportion* is better expreſſed by one and the ſame Quantity with the Sign after them. Thus, a, aa, aaa, $aaaa$, a^5, a^6, $\div$ &c. are Quantities in *Geometrical Proportion*. So alſo 2, 4, 8, 16, 32 $\div\!\div$, &c. are in *continual Proportion*.

7. This Character (:) ſignifies the Word *to*; and this (: :) ſignifies the Words *ſo is*. When they are join'd together thus, (: : : :) they are the Rule of Proportion; and being placed between Numbers or Quantities, (thus, $a : b :: d : e$) they are thus read

or

or expreſs'd, As *a* to *b*, *ſo is d* to *e*. Or thus, 4 : 6 : : 8 : 12,) is, As 4 to 6, *ſo is* 8 to 12, *&c.*

8. This Character (ϕ) is here uſed to ſignify *Tranſpoſition*, and ſhews that the Number or Quantity before which it is plac'd is in the next Line or Step tranſpos'd to the other Side of the Equation, with a contrary Sign.

9. This Character or Letter (Q.) is alſo here uſed, and ſignifies *by the Queſtion*, as you may ſee in the Work of the following Problems.

10. This Character ($\surd$ is a radical Sign, or Sign of the Square Root, and ſhews that the Number or Quantity before which it ſtands is to have its Square Root extracted.

11. This Character ($\sqrt[3]{}$) is the Sign of the Cube Root, and ſignifies the Extraction of it, as in the Square Root above.

SECT. IV.

Of COMPOUND SIGNS *or* CHARACTERS.

1. THIS ($\sqrt[3]{}\ a \times b + c$) ſignifies, that the Product of *a* and *b* added to *c* is to have its Cube Root extracted. The ſame of the Square Root.

2. This, ($\surd\ a \times b + c - d$) ſhews, that after the Quantity *d* is taken out of the Product of *a* and *b* more the Sum of *c*, the Remainder is to have its Square Root extracted.

3. This

3. This long Dafh (———) is often ufed to link or couple Quantities together for the better reading or underftanding them. Befides they are differently expreffed to what they are when it is wanting. Thus, $a \times b + c + d$ has quite a different Signification to what it has with the Dafh over it thus, $a \times \overline{b + c + d}$; for $a \times \overline{b + c + d}$, fignifies, that the Quantity a is to be multiplied by the Sum of b, c, and d; whereas, without the Dafh, it would fignify only that the Sum of the Quantities c and d is to be added to the Product of a into b. This will be beft underftood by Numbers.

☞ Let 6 reprefent a, 4 b, 8 c, and 12 d. Now $6 \times \overline{4 + 8 + 12} = 144$; but $6 \times 4 + 8 + 12$, will be but 44. So that the Difference on Account of the Dafh is 100, *&c. &c.* *Note further*,

This Dafh is often join'd to the Tail of the radiant Sign, or the Sign is continued longer, which is the fame; and according as how far the Dafh is extended it has a different Signification.

4. Thus ($\sqrt{m + \frac{bb}{4} + dd} - c$) fignifies, that the Quantity c is to be taken out of the Square Root of $m + \frac{bb}{4} + dd$.

5. But $\sqrt{m + \frac{bb}{4}} + dd - c$) fignifies, that only the Square Root of $m + \frac{bb}{4}$ is to be extracted, and then the Difference between the Quantities $dd - c$ to be added to it. The fame for the Cube Root.

6. This

6. This Character ($\pm$) ſignifies *more or leſs ſuch a Quantity*, and is uſed often in Extraction of Roots, compleating of Squares, *&c.*

7. ☞ Figures are frequently ſet over Quantities, to ſhew how often they are expreſſed, and to ſave the Trouble of repeating or ſetting down the Letters ſo often. Thus, b^4 ſignifies the ſame as if the Quantity b was written, expreſs'd, or ſet down four Times, thus, *bbbb*. So alſo x^8 is *xxxxxxxx*, *&c.*

CHAP.

CHAP II.

DIALOGUE V.

Between PHILOMATHES *and* TYRUNCULUS, *concerning* ADDITION, SUBTRACTION, MULTIPLICATION, *and* DIVISION *of* ALGEBRA.

Phi. ——————— (*Coming to visit* Tyrunculus.)
Tyr. ——————— (*Standing at his Door.*)

Phi. GOOD Morrow to you *Tyrunculus.*

Tyr. Kind *Philomathes*, your Servant; if I may be ſo free, where are you walking this Morning?

Why, *Tyrunculus*, to ſay the Truth, I came only for to ſpeak to you; it is ſome Time ago (if you remember) ſince I gave you a Paper, and I expected before now that you would have come for freſh Inſtructions and Examples; but your ſtaying ſo long began to make me think that you had given over the Thoughts of *Algebra* again, and had neglected to learn the *Signs* and *Characters* you had of me for that Purpoſe.

Tyr.

Tyr. I am ſorry I have given you Occaſion to think ſo; but the Reaſon of my not being with you before, is becauſe I have had ſome particular Buſineſs upon my Hands, and *that* you know muſt be minded: However, I have learnt them perfectly by Heart, and know the Meaning of them very well.

Phi. I am glad of it; and you intend, I hope, to apply them to Practice; do you not?

Tyr. To be ſure I do, as far as Leiſure from more material Things will allow of.

Phi. Well then, are you at Liberty to go Home with me now, I am no Ways engaged?

Tyr. If you inſiſt upon it I will; but I had much rather you would ſpend an Hour or two with me now you are here; you ſhall be heartily welcome to ſuch Entertainment as my little Houſe affords, and I ſhall eſteem it as an Inſtance of your Kindneſs. What ſay you?

Phi. I heartily thank you, *Tyrunculus.* Entertainment by Way of eating and drinking I regard not, any further than to ſatisfy the real Wants of Nature: It is the Converſation I value, and had rather pleaſe my Mind than my Appetite; therefore upon Promiſe that you will not put yourſelf to any Trouble, nor provide for me any other than *that* which you intended for yourſelf had I not dropt in, I will ſpend a few Hours with you.

Tyr. Upon Honour I will not.

Phi. Come then, *Tyrunculus*, let us be doing.

Tyr. With all my Heart; and pray what is the firſt Thing in *Algebra* that I am to begin with?

Phi. As you underſtand *Vulgar Fractions*, and know the *Signs* and *Characters* you ſay, the very firſt Thing that I ſhew you will be *Addition.*

SECT.

SECT. I.

ADDITION *of* ALGEBRA.

Tyr. HOW is *Addition* of *Algebra* performed?

Phi. The ſame as common *Addition*, provided the Signs be both *affirmative* or both *negative*; as you will ſoon find by the four following Caſes.

CASE I.

Of Simple Quantities *or* Integers, *having the ſame Sign.*

When the Quantities to be added have the ſame Sign, (viz. *both* + *or both* —) *then add all the* Co-efficients *or Numbers together,* (*if any there be*) *and place the Quantity after them, with the ſame Sign alſo before them.*

	Ex. 1.	Ex. 2.	Ex. 3.
Add	$+ a$	$+ 2b$	$- 5d$
	$+ a$	$+ b$	$- d$
	$+ a$	$+ 4b$	$- 7d$
Sum	$+ 3a$	$+ 7b$	$- 13d$

	Ex. 4.	Ex. 5.
	$+ 5aadd$	$- bcd$
	$+ 4aadd$	$- bcd$
	$+ 6aadd$	$- bcd$
Sum	$+ 15aadd$	$- 3bcd$

Do you underſtand theſe Examples?

Tyr. Yes, except the (b) that ſtands alone in *Example* 2. and ($-d$) in *Example* 3. for they have no Number or Co-efficient before them.

Phi. It is true they have not; but they are ſuppoſed (as all ſingle Quanties are) to have Unity or 1 placed before them, as you may ſee in *Example* 1. and *Example* 5. whoſe Sums are $3a$, and $-3\,bcd$.

Tyr. I underſtand you now very well.

Phi. You are then further to take Notice, that all Quantities that have not the *negative* Sign ($-$) placed before them, are ſuppoſed to have the *affirmative*: Or, in other Words, thus: When any Quantity has no Sign prefix'd to it, it is then an *affirmative* Quantity; thus, a is the ſame as $+a$, and $4\,b$ is $+4\,b$.

CASE 2.

Of SIMPLE INTEGERS *or* QUANTITIES, *having contrary Signs.*

When the given Quantities are alike, but have unlike Signs, then ſubtract the one Co-efficient *from the other, and place the* negative *or* affirmative *Sign to the Remainder, according to where the Exceſs lies; that is, if the* negative *Quantity has the greater* Co-efficient, *then the Remainder will be* negative *and muſt have the Sign* ($-$); *but if the* affirmative *be the larger, then will the Remainder have the Sign* ($+$) *and will be the Sum of the ſaid Quantities.*

Ex. 1.	Ex. 2.	Ex. 3.
$4\,bb$	$-7\,bb$	$14\,aabb$
$-3\,bb$	$3\,bb$	$-43\,aabb$
$+\ \ bb$	$-4\,bb$	$-29\,aabb$

Ex.

Ex. 4.	Ex. 5.
— *abcd*	42 *bbdd*
9 *abcd*	— 42 *bbdd*
+ 8 *abcd*	(oo). *

Do you underftand thefe Examples?

Tyr. I underftand all but the 5*th.* for I cannot at prefent conceive, that + 42 added to — 42 can be equal to Nothing, I fhould think rather, that fubtracting them they would be equal to Nothing.

Phi. That is your Miftake, for their Difference is 84, (as you will fee *Cafe* 2. in *Subtraction*); becaufe the *negative* Sign makes void the *affirmative.*

Tyr. I afk Pardon, but I do not rightly apprehend it.

Phi. I think you are a little dull now. Do you not remember that I told you, (*Sect.* 3. *Dialogue* 4.) that this Sign (—) fignifies a Want or Deficiency, fo many Times *lefs* than Nothing as the Figures after it exprefs?

Tyr. Yes I do.

☞ *Phi.* Obferve then, Suppofe that you ftood indebted to a Perfon 42 *£.* and had no Effects of any Sort to pay the Debt, then it is plain you would be 42 Times worfe than Nothing; that is, have 42 Times lefs than a real Property of your own: Now fuppofe a Friend fhould give you 42 *£.* to pay off the Debt, and you do fo, ftill it is plain you would have Nothing in Hand to begin the World again with: Confequently then + 42 added to — 42 = * or o.

* Left the Learner fhould miftake the Remainder in this Example, and take it for fome Quantity, I thought proper to put it in a Parenthefis, to fignify it ftands for a Cypher only, and not for any Quantity. See *Dial.* 9. *Obfervation* 2.

Tyr. I am very thankful, *Philomathes*, for ſo plain a Demonſtration.

☞ So I perceive then, that a *negative* Quantity added to an *affirmative* one, is the ſame as two *affirmative* Quantities ſubtracted from each other. Are they not?

Phi. The very ſame. (See *Caſe* 2. *Ex.* 5. in *Subtraction.*)

Tyr. Thus far then I am pretty perfect; but how muſt I manage when the Quantities are many in Number, and have different Signs?

☞ *Phi.* Very eaſily. Firſt collect all the Quantities that have one and the ſame Sign into one Sum, ſo you will have two Quantities at laſt to be added as above.

Tyr. Pray add $14\,axx - 5\,axx + 8\,axx - axx - 41\,axx + 39\,axx$ together?

Phi. Obſerve then, I collect all the Quantities having one and the ſame Sign together, *viz.* $14\,axx + 8\,axx + 39\,axx$, and theſe are equal to $61\,axx$; then $-5\,axx - axx - 41\,axx = -47\,axx$; then $-47\,axx$ added to $+61\,axx$, as before directed, $= 14\,axx$ *Anſwer*, the Sum of all the Quantities. Now I will try you with a Queſtion.

EXAMPLE 7.

Add $4\,aaa + 9\,aaa - 12\,aaa - 4\,aaa - 19\,aaa + 14\,aaa + 6\,aaa - 9\,aaa$ *together.*

Tyr. Nothing eaſier. Firſt, $4\,aaa + 9\,aaa + 14\,aaa + 6\,aaa = +33\,aaa$; and $-12\,aaa - 4\,aaa - 19\,aaa - 9\,aaa = -44\,aaa$; *then* $-44\,aaa$ added to $+33\,aaa = -11\,aaa$ *Anſwer.*

Phi You are right, but ſtill you have taken unneceſſary Trouble.

Tyr. Wherein?

Phi.

☞ *Phi.* Why did not I tell you that the *negative* Sign $-$ deſtroys the *affirmative* $+$; therefore as you have $4\,aaa + 9\,aaa$, and alſo $-4\,aaa - 9\,aaa$ in the Queſtion, you needed not to have meddled with them at all, only add the reſt of the Quantities, and you will find them to be $+20\,aaa$ and $-31\,aaa$, whoſe Sum is $-11\,aaa$, as above.

Tyr. I was a little wanting indeed in this Reſpect.

Phi. To be ſure it ſaves Trouble; for ſuppoſe I were to add $abb + 6\,abb - 9\,abb - abb + 4\,abb - 6\,abb + 9\,abb - 3\,abb$ together, I have only $-3\,abb$ to add to $+4\,abb$, (for the reſt deſtroy each other by contrary Signs) and their Sum is $+abb$, the Sum of all.

Tyr. I ſhall take Notice of it; but ſuppoſe the Quantities to be added are not alike, nor the Signs neither, how then?

Phi. You will ſee by the following *Caſe.*

CASE 3.

Of SIMPLE CONTRARY QUANTITIES.

When the Quantities to be added are unlike, whether they have Co-efficients *or not, ſet them one after another, without any Alteration of the Signs, and this will be the proper Sum.*

Tyr. This is eaſy indeed; then if I were to add $3\,bc + aa + g - d$ together, I imagine their Sum is the ſame; *viz.* $3\,bc + aa + g - d$: Is it not?

Phi. You are very right. See the following *Example.*

EXAMPLE I.

$$\begin{array}{l} -5\,bc \\ +4\,da \\ -\;\;xx \\ -4\,cg \\ \hline \end{array}$$

Sum $-5\,bc + 4\,da - xx - 4\,cg$

Ex-

EXAMPLE 2.

$$\begin{array}{l} -\ 57\ gb \\ +\ \ 4\ m \\ -\ \ \ \ ggg \\ +\ 17 \\ \hline \end{array}$$

Sum $57\,gb + 4\,m - ggg + 17$

CASE 4.

Of COMPOUND INTEGERS *or* QUANTITIES.

This depends upon the three preceding Cases: For if the Quantities are alike, and have the same Sign, then add them together by Case 1. *but if they have contrary Signs, collect all such together as have one and the same Sign, and subtract them from each other, setting the Sign where the Excess lies (according to* Case 2.) *be it* + *or* —. *But if the Quantities and Signs be both contrary, then, (by* Case 3) *set them one after another, without altering any of the Signs, and you have the Total.*

Ex. 1.	Ex. 2.
Add $4x + b$	$14x - 9$
$x + 4b$	$-\ 9x + 14$
Sum $5x + 5b$	$+\ 5x + 5$

EXAMPLE 3.

$$\begin{array}{l} \text{Add } 4\,bc + x - 18 \\ \quad\ \ 9\,bb - xb + 9 \\ \hline \end{array}$$

Sum $4\,bc + x + 9\,bb - xb - 9$

More

More EXAMPLES.

EXAMPLE 4.

$$\begin{array}{rl} & 4\,abcd + 6\,bc + 9\,abd - 9 \\ - & abcd - 4\,bc - 5\,abd \\ + & 2\,abcd - 3\,bc + 2\,abd \\ - & abcd + bc + abd + 7 \\ \hline \text{Sum} & 4\,abcd + 7\,abd - 2 \end{array}$$

EXAMPLE 5.

$$\begin{array}{rl} & a + 4\,b + g + ee + 7\,xx + y \\ & x + bb + nn + 4\,ff - 12\,y \\ & -a - g + 4\,ee + 4\,bb - xx - ff \\ & -5\,ee - 3\,ff + 5\,y - nn + 5 - zz \\ \hline \text{Sum} & 4\,b + 6\,xx + 5\,bb + x - 6\,y + 5 - zz \end{array}$$

Tyr. You have made Ufe of all the Cafes I fee in thefe *Examples.*

Phi. I do it on Purpofe to ferve you; and do you think you are perfect in *Addition?* Look at *Example* 5 once more if you be not.

Tyr. I think I am indeed, Sir.

Phi. If fo, we will pafs directly to *Subtraction.*

SECT. II.

SUBTRACTION *of* ALGEBRA.

Tyr. I Am afraid *Subtraction* will puzzle me, for I remember I was more fet in this Rule in *Vulgar Fractions* than in any of the other.

Phi.

Phi. Be not at all discouraged, for you will presently do it I am sensible, if you take Care to mind the Rules; for I shall proceed the same as in *Addition.*

CASE I.

Of SIMPLE INTEGERS, *having the same Sign.*

When you have any Quantity given to be subtracted from another, then change the Sign of the Quantity to be subtracted into the contrary Sign; that is, if it be + *make it* —, *and if* — *make it* +, *and then add them by the first* Case *of* Addition, *and you have the true Difference: Or, If the Quantities have one and the same Sign, then subtract the* Numbers *or* Co-efficients *as in common* Subtraction, *observing to place the Sign belonging to them to the Remainder.*

Ex. 1.	Ex. 2.	Ex. 3.
From $3a$	$-21b$	$8bcd$
Take a	$-9b$	$3bcd$
Ans. $2a$	$-12b$	$5bcd$

EXAMPLE 4.

From $-24abd$
Take $-9abd$

Ans. $-15abd$

CASE 2.

Of SIMPLE INTEGERS, *having contrary Signs.*

When the Quantities to be subtracted have the negative Sign, *and the other the* affirmative, *then add them together as in common* Addition, *and you have their true*

true Difference; to which place the affirmative Sign; *but if the Quantity to be subtracted have the* affirmative Sign, *and the top Quantity the* negative, *you are to add as before, but then place the* negative Sign *to their Sum, and it is the true Difference.*

	Ex. 1.	Ex. 2.	Ex. 3.
From	$+ 4\,bb$	$- 7\,bb$	$14\,aabb$
Take	$- 3\,bb$	$+ 3\,bb$	$- 43\,aabb$
Diff.	$+ 7\,bb$	$- 10\,bb$	$57\,aabb$

	Ex. 4.	Ex. 5.
From	$-\,abcd$	$+ 42\,bbdd$
Take	$9\,abcd$	$- 42\,bbdd$
Difference	$- 10\,abcd$	$+ 84\,bbdd$

EXAMPLE 5. *varied.*

From	$42\,bbdd$	$+ 42\,aabb$	$- 42\,bbdd$
Take	$42\,bddd$	$- 42\,aabb$	$+ 42\,bbdd$
Diff.	(0)	$+ 84\,aabb$	$- 84\,bbdd$*

CASE 3.

Of COMPOUND QUANTITIES.

There is no Difference in the Work of these and simple Quantities, only observe to place the Sign between the Difference according as is required.

	Ex. 1.	Ex. 2.
From	$a + b$	$a - b$
Take	$a - b + 9$	$- a + b - 9$
Ans.	$+ 2b - 9$	$2a - 2b + 9$

Ex-

* See *Case 2d. Ex.* 5. in *Addition.*

EXAMPLE 3.

From $\quad 7\,aa + \quad b - 7\,cc + 14$
Take $- \quad aa - 4\,b + 2\,cc - 10$

Anſ. $+ 8\,aa + 5\,b - 9\,cc + 24$

Tyr. Mighty pretty, and withal very eaſy. Pray how are unlike Quantities ſubtracted?

Phi. I will tell you directly.

CASE 4.

Of CONTRARY *or* UNLIKE QUANTITIES.

When you are to ſubtract unlike Quantities, let them be either ſimple *or* compound, *you have no more to do, but only to ſet the Quantities to be ſubtracted directly after the Quantity you ſubtract from, firſt obſerving to change or alter their Signs; thus have you their true Difference*

EXAMPLE 1.

From $a + b - 5\,c$
Take $g - d + 9$, by changing the Signs.

Anſ. $a + b - 5\,c - g + d - 9$

EXAMPLE 2.

From $42\,bc + 24\,d - gg$
Take $17\,b - 14\,xx - bb - 15$

Anſ. $42\,bc + 24\,d - gg - 17\,b + 14\,xx + bb + 15$

Tyr. Quite plain indeed: Pray what is next?

Phi. Now, *Tyrunculus*, we are come to *Multiplication.*

SECT.

SECT. III.

MULTIPLICATION *of* ALGEBRA.

Tyr. I Think that *Subtraction* is eaſier than I thought for.

Phi. Nothing like Delight and Application, *Tyrunculus*, theſe make Things eaſy.

Tyr. It is very true; but I am afraid of *Multiplication.*

Phi. Moſt Learners dread a freſh Rule, though they wiſh too to be trying at it: But what ſhould be the Reaſon for ſuch Fear, ſince the ſame Care will conquer one as well as the other.

Tyr. Pray how is *Multiplication* performed?

Phi. There are three Things to be obſerved in this Rule, *viz.*

1. *When the Quantities have the ſame Sign.*
2. *When they have contrary Signs.*
3. *When they have Co-efficients or Factors.*

In all which Caſes you are carefully to obſerve, that when the Signs are both *affirmative*, or both *negative*, the Product will be *affirmative*; but when one is *affirmative*, and the other *negative*, the Product will be *negative:* For $+ \times +$, or $- \times -$, produce $+$; but $+ \times -$, or $- \times +$, produce leſs.

CASE I.

Of QUANTITIES *having the ſame Sign.*

If the Quantities have no Factors *or* Co-efficients, *then only join the Letters repreſenting ſuch Quantities, one by the Side of the other, and place the Sign* $+$ *before*

fore them; but if they have Factors, *multiply them as in common* Multiplication, *placing the Product before the Quantities.*

	Ex. 1.	Ex. 2.	Ex. 3.
Mult.	a	$4b$	$-ab$
by	b	c	$-d$
Product	ba	$4bc$	$+abd$

	Ex. 4.	Ex. 5.
Mult.	$-6x$	$14bb$
by	$-2g$	$2b$
Product	$+12xg$	$28bbb$

Tyr. This is eafy enough indeed.

Phi. Nothing eafier, therefore we will pafs to *Cafe* 2.

CASE 2.

To MULTIPLY *contrary Signs.*

	Ex. 1.	Ex. 2.	Ex. 3.
Mult.	b	ba	$-4bcd$
by	$-a$	$-b$	$+6b$
Product	$-ab$	$-bba$	$-24bbcd$

EXAMPLE 4.

Multiply	$8xxb$
by	$-4d$
Product	$-32xxbd$

Tyr.

Tyr. There needs no more Examples. Pray what is the next?

Phi. CASE 3.

Of COMPOUND QUANTITIES.

Multiply every particular Quantity in the Multiplicand, *by each Member or Part of the* Multiplier, *as you do in common* Multiplication; *then add them together by the Rules of* Addition, *and the Work is done.*

	Ex. 1.	Ex. 2.
Mult.	$ab + c$	$4\,b - cd$
by	d	$4\,b$
Product	$abd + dc$	$16\,bb - 4\,cdb$

EXAMPLE 3.

Mult.	$9\,b - 8 + 5\,x$
by	$-\,3\,d$
Product	$-\,27\,bd + 24\,d - 15\,xd$

More EXAMPLES *of* COMPOUND QUANTITIES.

	EXAMPLE 4.	EXAMPLE 5.
Mult.	$a + b$	$a - b$
by	$a + b$	$a - b$
	$aa + ab$	$aa - ab$
	$+ ab + bb$	$- ab + bb$
Prod.	$aa + 2\,ab + bb$	$aa - 2\,ab + bb$

EXAMPLE. 6.

$$\begin{array}{lll} \text{Mult.} & a + b & \\ \text{by} & a - b & \\ \hline & aa + ab & \\ & \quad - ab & - bb \\ \hline \text{Product} & aa & - bb \end{array}$$

EXAMPLE 7.

$$\begin{array}{ll} \text{Mult.} & ab + cd - 6 \\ \text{by} & c + d \\ \hline & abc + ccd - 6c \\ & \quad + abd + cdd - 6d \\ \hline \text{Prod.} & abc + ccd - 6c + abd + cdd - 6d \end{array}$$

EXAMPLE 8.

$$\begin{array}{ll} \text{Mult.} & 2b + cd - 4 \\ \text{by} & -7x + 3 \\ \hline & -14bx - 7cdx + 28x \\ & \quad + 6b + 3cd - 12 \\ \hline \text{Prod.} & -14bx - 7cdx + 28x + 6b + 3cd - 12 \end{array}$$

Ex-

EXAMPLE 9.

$$\begin{array}{lllllll} \text{Mult.} & a + b + c \\ \text{by} & a + b + c \\ \hline & aa + ab + ac \\ & \quad + ab + bb + bc \\ & \qquad + ac + bc + cc \\ \hline \text{Prod.} & aa + 2\,ab + 2\,ac + 2\,bc + bb + cc \end{array}$$

EXAMPLE 10.

$$\begin{array}{ll} \text{Mult.} & a + b + c \\ \text{by} & a - b - c \\ \hline & aa + ab + ac \\ & \quad - ab - bb - bc \\ & \quad - ac - bc - cc \\ \hline \text{Product} & aa - bb - 2\,bc - cc \\ \hline \end{array}$$

Tyr. I underſtand the Examples very well; I ſee in this laſt Example, that $+ ab - ab$, $+ ac$ and $- ac$ deſtroy each other when you come to add them together.

Phi. Very well obſerved, *Tyrunculus*; this gives me Reaſon to hope you are perfect in what you have ſeen done.

Tyr. I hope ſo. But pray, *Philomathes*, let me aſk a Queſtion. I am not yet fully ſatisfied why $+ \times -$ ſhould produce $-$, and that $- \times -$ ſhould produce $+$; have you no other Way to demonſtrate it but barely telling me ſo?

Phi. To be ſure, it is eaſily proved as follows. Suppoſe I take 4 Quantities, and make them equal to

 any

any 4 Numbers, *viz.* Suppose $b = 12$, $c = 4 = d$ $= 8$, *and* $f = 6$. Then place them in some such like Order as follows:

$$\text{Subtract}\left\{\begin{matrix} b = 12 \\ c = 4 \end{matrix}\right. \text{ and } \left.\begin{matrix} d = 8 \\ f = 6 \end{matrix}\right\}\text{Subtract}$$

Then $b - c = 8$, and $d - f = 2$

Now $\overline{b-c} \times \overline{d-f} = 8 + 2 = 16$, the Product of their correspondent Numbers 8 and 2. Therefore $\overline{b-c} \times d - f$ will be $= db - dc - fb + fc$, *viz.* $= 16$, if the Work be right.

PROOF.

$$\left.\begin{matrix} \text{First, } b \times d = 12 \times 8 = 96 \\ \text{And } c \times f = 4 \times 6 = 26 \end{matrix}\right\}\text{Add}$$

$$\text{Therefore } bd + cf = 120$$

Again,

$$\left.\begin{matrix} d \times c = 8 \times 4 = 32 \\ f \times b = 6 \times 12 = 72 \end{matrix}\right\}\text{Add}$$

That is, $dc + fb = 104$, which taken from 120, leaves 16; that is, $bd + cf - dc - fb = 120 - 104 = 16$, as above. Consequently therefore $+ \times -$ produces $-$, and $+ \times +$, or $- \times -$ produces $+$, Q. E. D.

Tyr. I cannot say I understand it by looking it over so quick, but I will consider of it another Time. In the mean Time, pray tell me what is the Meaning of Q. E. D.?

Phi. They are often used in the *Mathematicks*, and set at the End of a Demonstration or Proof of any Problem, and signify, that the Thing given is not

not only done, but demonſtrated and proved, and is thus read, Q. E. D. *which was to be proved.*

Tyr. I heartily thank you. Pray have you any Thing more to ſhew me in this Rule?

Phi. Nothing more, only leave a Sum for you to work at your Leiſure, which I would have you try at.

EXAMPLE II.

Mult. $3\,xx + 4\,bb - 2\,d$
by $2\,xx - 3\,bb - \quad d$

Prod. $6x^4 - xxbb - 7\,dxx - 12\,b^4 + 2\,bbd + 2\,dd.$

SECT. IV.

DIVISION *of* ALGEBRA.

Tyr. I Fear you will find me but a dull Scholar at *Diviſion*; for as common *Diviſion* is harder than the other three Rules, I think of Courſe *Diviſion* of *Algebra* muſt.

Phi. I confeſs it is ſomething more difficult; but what of all this? *Labour overcomes all Things.* However, it is eaſier perhaps than you imagine.

Tyr. How is *Diviſion* performed?

☞ *Phi.* In *Diviſion* are three Caſes; in all which you are to obſerve, as in *Multiplication*, that if the Signs be alike in the *Diviſor* and *Dividend*, the *Quotient* will be *affirmative*; but if unlike, the *Quotient* muſt have the *negative* Sign.

CASE I.

Of QUANTITIES *having the ſame Sign.*

If the Quantities have Co-efficients *belonging to them, then divide the one by the other, as in common* Diviſion, *and place the Quantities in the Quotient. But if the Quantities have no* Co-efficients, *then ſet the* Dividend *a-top, and the* Diviſor *under it,* Fraction-wiſe, *and if you find the ſame Letters or Quantities in both, cancel or caſt away ſuch Letters, and the Remainder will be the true Quotient or Anſwer required. Thus, ſuppoſe I were to divide* dcb *by* db, *I ſet it thus,* $\frac{dcb}{db}$; *and becauſe I find* db *both in the* Dividend *and the* Diviſor, *I expunge or cancel them, and then only* c *remains, which is the Quotient or Anſwer.*

Ex. 1.

Divide by $\frac{bbdc}{bc}$ *Anſ.* bd.

Ex. 2.

$\frac{abc}{da}$ *Anſ.* $\frac{bc}{d}$

EXAMPLE 3.

Divide by $\frac{xxddg}{xbgd}$ *Anſ.* $\frac{xd}{b}$

	Ex. 4.	Ex. 5.	Ex. 6.
Divide	$15\ xb$	$-\ 64\ bd$	$-\ 4\ abc$
by	$5\ b$	$-\ 8\ b$	$-\ 4\ abc$
Anſ.	$3\ x$	$+\ 8\ d$	$+\ 1$

CASE 2.

Of CONTRARY SIGNS and QUANTITIES.

Divide the Quantities *and* Co-efficients *as before, and to the Quotient annex the Sign* —, *and the Work is done.*

	Ex. 1.	Ex. 2.	Ex. 3.
Divide	ab	$-15\,xb$	$64\,bdc$
by	$-\,a$	$+\,5\,b$	$-\,8\,bd$
Anſ.	$-\,b$	$-\,3\,x$	$-\,8\,c$
Proof	ab	$-15\,xb$	$64\,bcd$

EXAMPLE 4.

Divide	$4\,abc$
by	$-\,4\,abc$
Anſ.	$-\,1$
Proof.	$4\,abc$

Note. *When the* Sign *and* Quantities *are quite unlike, and the* Co-efficients *cannot be divided, then ſet them over one another* Fraction-wiſe *and you have the Anſwer.*

	Ex. 5.	Ex. 6.	Ex. 7.	Ex. 8.
Divide	ab	$14\,abg$	$xxbd$	$15\,xn$
by	g	$5\,xx$	ac	$12\,ag$
Anſ.	$\frac{ab}{g}$	$\frac{14\,abg}{5\,xx}$	$\frac{xxbd}{ac}$	$\frac{15\,xn}{12\,ag}$

Tyr.

Tyr. I underftand it very well; but how do you divide *compound Quantities?*

Phi. The fame as in *Multiplication*, by going through every Member of the *Dividend* and *Divifor* according to the Order of common *Divifion*.

CASE 3.
Of COMPOUND QUANTITIES.

Rule 1.

Proceed in all Refpects as before, due Regard being had to the Signs, *and you have your Defire.*

Ex. 1.	Ex. 2.
Divide $ada + abcd$	$12\,dc - 15\,dx + 4\,ca - 5\,xa$
da	$3\,d + a$
Anf. $a + bc$	$4\,c - 5\,x$

Tyr. Let me afk you one Queftion?

EXAMPLE 3.

$$\begin{array}{ll} \text{Divide} & 4\,abc - 24\,aabb - 32\,bad \\ \text{by} & \quad\quad - 4\,ab \\ \hline \end{array}$$

Phi. Anf. $-1\,c + 6\,ab + 8\,d$

Tyr. Very well; but pray is not *Divifion* wrought fometimes at large, or do no Queftions require it?

Phi. It is frequently fo wrought, and I could have done the laft Example fo if I would.

Tyr. Pray do, it may perhaps give me a better Idea of *Divifion* than I have at prefent.

Phi. Obferve then.

2. *When you have many* Quantities, *then proceed as in* Divifion *of whole Numbers, by feeing how many Times the* Divifor *is contained in the* Quantities *of the* Dividend,

Dividend, *placing it in the* Quotient; *then multiply the* Divifor *by the* Quotient *and place it under the* Dividend; *and fubtract it therefrom, and to the Remainder bring down the next* Quantity *or* Quantities *in the* Dividend, *and thus proceed till the whole Operation is performed.*

EXAMPLE 3. *The long Way.*

$$
\begin{array}{llll}
-4ab) & 4abc-24aabb-32bad & (-1c+6ab+8d\ \textit{Anf.} \\
 & +4abc & & \\
\hline
 & \quad -24aabb & & \\
 & \quad -24aabb & & \\
\hline
 & \qquad -32bad & & \\
 & \qquad -32bad & & \\
\hline
 & \qquad\quad 0 & &
\end{array}
$$

Tyr. I like this very well, and it appears much plainer to me than the other. Pray give me one more Example?

Phi. Suppofe it were required to divide $bx+bd+cx+cd-xe-de$ by $x+d$; it will ftand as follows.

EXAMPLE 4.

$$
\begin{array}{llll}
x+d) & bx+bd+cx+cd-xe-de & (b+c-e\ \textit{Anf.} \\
 & bx+bd & & \\
\hline
 & 0 \quad cx+cd & & \\
 & \quad\ \ cx+cd & & \\
\hline
 & \qquad 0 \quad -xe-de & & \\
 & \qquad\quad\ \ -xe-de & & \\
\hline
 & \qquad\qquad 0 & &
\end{array}
$$

Ex-

EXAMPLE 5.

$$
\begin{array}{llll}
3x-6)\ 6xxxx - 96\ (2xxx+4xx+8x+16\ \textit{Anf.} \\
 6xxxx-12xxx^{*} \\
\hline
 0 \quad +12xxx-96 \\
 +12xxx-24xx \\
\hline
 0 \quad +24xx-96 \\
 +24xx-48x \\
\hline
 0 \quad +48x-96 \\
 48x-96 \\
\hline
 0
\end{array}
$$

Tyr. I like this quite well indeed; but I could not have thought that ſo ſhort a *Dividend* would have produced ſo many *Quantities* in the *Quotient*.

Phi. That is eaſy to perceive; becauſe $*-12xxx$ is not found in the *Dividend*. I change the Sign (which is ſubtracting it) and bring it down for a new *Dividend* or *Remainder*, and it will be $+12xxx$. I do the ſame alſo with $-24xx$ and $-48x$. See the

PROOF.

$$
\begin{array}{l}
2xxx+4xx+8x+16 \\
3x-6 \\
\hline
6xxxx+12xxx+24xx+48x \\
 -12xxx-24xx-48x-96 \\
\hline
6xxxx-96.
\end{array}
$$

Tyr. 'Tis right ſure enough.

Phi.

Phi. I ſhall give you then but one Example more.

EXAMPLE 6.

$$\begin{array}{l} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\ddagger \qquad\qquad\qquad\qquad\qquad (-72 \\ 5b-9)20ab+5xxb-60b+30bc-5bd+40b-36a-9xx+108-54c+9d- \\ \quad 20ab--36a^{*} \qquad\qquad\qquad\qquad\qquad\qquad (4a+xx-12+6c-d+8\ \mathit{Anſ.} \\ \quad \overline{\qquad\qquad} \\ \qquad +5xxb-60b \\ \qquad +5xxb-9xx \\ \qquad \overline{\qquad\qquad} \\ \qquad\quad -60b+30bc \\ \qquad\quad --60b+108 \\ \qquad\quad \overline{\qquad\qquad} \\ \qquad\qquad +30bc--5db \\ \qquad\qquad +30bc-54c \\ \qquad\qquad \overline{\qquad\qquad} \\ \qquad\qquad\quad -5db+40b \\ \qquad\qquad\quad -5db+9d \\ \qquad\qquad\quad \overline{\qquad\qquad} \\ \qquad\qquad\qquad +40b-72 \\ \qquad\qquad\qquad +40b-70 \\ \qquad\qquad\qquad \overline{\qquad\qquad} \\ \qquad\qquad\qquad\qquad 0 \end{array}$$

Tyr. I do not underſtand this, that $-{}^{*}36a$ ſhould ſtand under $5xxb$: Pray how do you ſubtract them?

Phi. I do not ſubtract it from $5xxb$, though it ſtands under it, but from $-\ddagger 36a$ further on in the Dividend, and ſo on for the reſt.

Tyr. Then it Matters not whether the Quantities ſtand under each other I imagine; does it?

Phi. Not at all, as long as you can but find the ſame Quantity any where in the Dividend; and you would do well in ſuch long Sums to make a particular Mark againſt ſuch Quantities as you have taken down or done with, as you do in common *Diviſion.*

Tyr. I'll remember it; but ſuppoſe there ſhould be any Remainder (for I imagine the Quantities will not always fall out even) how then do you manage it?

Phi.

Phi. The ſame as in common *Diviſion*, by placing it over the Diviſor *Fraction-wiſe.* Thus, ſuppoſe I were to divide $xx - bb + xc$, by $x + b$; I find the Quotient to be $x - b$, and there remains xc; which I place over the Diviſor, and the Anſwer ſtands thus, $x - b \frac{xc}{x+b}$

See the Work.

$$\begin{array}{rl} x+b) & xx - bb + xc \ (x - b \ \frac{xc}{x+b} \ \textit{Anſ.} \\ & xx + xb \\ & \overline{\quad - xb - bb + xc} \\ & \quad - xb - bb \\ & \overline{\quad\quad * \quad * \quad xc} \end{array}$$

Tyr. Have you any Thing more to offer in *Diviſion?*

Phi. I think there is no Occaſion for any more Examples.

Tyr. Pray what comes next?

Phi. According to the Order of *Arithmetic* the Rule of *Proportion* ſhould follow, but I ſhall ſpeak of this under *Dialogue* 7. and ſhew you firſt the Nature of *Algebraic Fractions*; though one would think there is no great Occaſion, ſince I have been ſo particular in *Vulgar Fractions*, in which, if you are perfect, you cannot miſs to underſtand the *Algebraic*, which are done one and the ſame Way, only with Letters inſtead of Numbers; and this can be no great Difficulty, ſince Numbers are only repreſented by ſuch Letters, as may be ſeen in the following *Dialogue.*

DIA-

DIALOGUE VI.

SECT. I.

Of ALGEBRAIC FRACTIONS, *and first of* REDUCTION.

CASE 1*.

To reduce a mixt Quantity to an improper ALGEBRAIC FRACTION.

MULTIPLY *the whole Quantity by the Denominator of the* Algebraic Fraction, *and to the Product add the Numerator.*

EXAMPLE 1.

Reduce $a\frac{b}{x}$ *to an* improper Fraction.

$$a\frac{b}{x}$$

$$\frac{xa+b}{x}\ \textit{Ans.}$$

EXAMPLE 2.

Reduce $a+b\frac{c}{d}$ *to an* improper Fraction.

Ans. $a+b\,\frac{c}{d}+d=\frac{da+db+c}{d}$

K Tyr.

* Compare this with *Case* 1. in *Vulgar Fractions.*

Tyr. I have ſeen ſeveral Books of *Algebraic Fractions*, but I do not remember any ſuch Examples as theſe: Are they neceſſary?

Phi. Certainly they are, and that you will ſee if you do but try the ſame by any Figures you pleaſe to make equal to the Quantities; and this will be ſome Help to you, and give you a Notion of an Equation.

In *Example* 1. let $a = 5$, $b = 3$, $x = 6$, then

$$a\frac{b}{x} = 5\frac{3}{6}$$

$$\frac{xa + b}{x} = \frac{33}{6}$$ *Anſ.* or rather $\frac{30 + 3}{6} = \frac{33}{6}$.

For $x \times a = 6 \times 5 = 30; + b = 3, = \frac{33}{6}$. Q. E. D. &c. &c.

Tyr. I think it is neceſſary indeed, as you ſay.

Phi. And the Beauty of it is, the very next Caſe proves it; for I ſhall take the ſame two Examples.

CASE 2*.

To reduce an IMPROPER FRACTION *to a* MIXT QUANTITY.

This is only the Reverſe of the former, for you have no more to do but to divide the Numerator by the Denominator, and it is done.

EXAMPLE I.

Reduce $\frac{xa + b}{x}$ *to its* equivalent mixt Numbers.

$$x)\ xa + b\ (a\frac{b}{x}\quad \textit{Anſ.}$$
$$xa$$
$$\overline{0 + b}$$

Ex-

* Compare this with *Caſe* 2. in *Vulgar Fractions*.

EXAMPLE 2.

Reduce $\frac{da + db + c}{d}$ *to its* equivalent mixt Numbers.

$$d)\ da + db + c\ (a + b\,\tfrac{c}{d}\quad \textit{Anf.}$$

$$\begin{array}{l} da \\ \hline 0 + db \\ \quad\ \ db \\ \hline \quad\ \ 0 + c \end{array}$$

CASE 3*.

To reduce a whole QUANTITY *to an* ALGEBRAIC FRACTION.

Multiply the given Quantity by any other Quantity, and place the Product for a Numerator, and the Quantity you multiplied by for a Denominator, and it is done.

EXAMPLE 1.

Reduce b *to an* Algebraic Fraction, *having* dc *for its Denominator.*

$b \times dc = bdc$ *Anf.* $\frac{bdc}{dc} = b$ by the next *Cafe.*

EXAMPLE 2.

Reduce a *to an* Algebraic Fraction, *having* x + f *for its Denominator.*

Firft $a \times x + f = ax + af$ *Anf.* $\frac{ax + af}{x + f}$. Do you underftand it?

* Compare this with *Cafe* 3. in *Vulgar Fractions.*

Tyr. I cannot miſs it; you need not give any more Examples.

Phi. I ſhall not; but you ſhall ſee how that this Caſe is only the Reverſe of the next. We ſhall take the ſame two Examples.

CASE 4*.

To abbreviate an ALGEBRAIC FRACTION.

Divide the Numerator by the Denominator, that is, expunge or caſt away ſuch Quantities as are found in both, and you have your Deſire.

EXAMPLE 1.

Reduce $\frac{bdc}{dc}$ *to its loweſt Terms.*

$\frac{bdc}{dc} \div$ (divided by) $dc = b$. See *Example* 1. in laſt *Caſe*.

EXAMPLE 2.

Reduce $\frac{ax + af}{x + f}$ *to its loweſt Terms.*

$\frac{ax + af}{x + f} \div x + f = a$. See *Example* 2. laſt *Caſe*.

So alſo $\frac{20\ aab}{60\ ba} + \frac{8\ bbd}{40\ bdgc} = \frac{1\ a}{3} + \frac{1\ b}{5\ gc}$.

☞ As *Fractions* are abbreviated by *Diviſion*, it is often cuſtomary to put Unity under ſuch Abbreviations when the Denominator is caſt away; that is, if the Anſwer be a whole Quantity, put the Figure 1 under it: Thus $a = \frac{a}{1}$, and $a + x + 4$, expreſs'd like a *Fraction*, is, $\frac{a + x + 4}{1}$

CASE

* Compare this with *Caſe* 4. in *Vulgar Fractions*.

CASE 5*.

To reduce Quantities of unequal Denominators to ALGEBRAIC FRACTIONS, *having a common Denominator.*

Multiply all the Denominators continually for a common Denominator, and every Numerator into all the Denominators except its own, which ſhall be new Numerators.

Tyr. I think I can do this directly.

Phi. No Doubt of it, for it is the ſame as *Caſe* 7. in *Reduction* of *Vulgar Fractions.*

EXAMPLE I.

Reduce $\frac{a}{b}$, $\frac{c}{d}$, *and* $\frac{e}{f}$ *to* Fractions *having a common Denominator.*

Tyr. Firſt then, $b \times d \times f = bdf$ for a common Denominator. Then, $a \times d \times f = adf$ N. N. Again, $c \times b \times f = cbf$ N. N. And laſtly, $e \times d \times b = edb$ N. N. So are adf, cbf, edb, new Numerators to be placed over the common Denominator bfd, and will ſtand as follows:

$$Anſ.\ \left.\begin{array}{l} \frac{adf}{bdf} = \frac{a}{b} \\ \frac{cbf}{bdf} = \frac{c}{d} \\ \frac{edb}{bdf} = \frac{e}{f} \end{array}\right\} \text{by } Caſe\ 4.$$

Phi. You are very right, *Tyrunculus*, and I am glad to ſee you ſo tractable. You ſee therefore that the

* Compare this with *Caſe* 7. in *Vulgar Fractions.*

the Order of *Algebraic Fractions* is the ſame as *Vulgar*.

Tyr. Yes, I perceive it, and I find your Words true now, that to underſtand *Vulgar Fractions* well ſaves a great deal of Trouble, that muſt unavoidably happen to thoſe that are ignorant of them. Pray what comes next?

Phi. I have here ſhewn you three Caſes more than are in general taken Notice of, that you might ſee the Relation that *Algebraic Fractions* bear to *Vulgar.* I ſhall therefore ſhew you now how to add them together.

SECT. II.

ADDITION *of* VULGAR FRACTIONS.

Tyr. AS *Reduction* of *Algebraic Fractions* is like *Vulgar*, I imagine that *Addition* is done much after the ſame Manner alſo: Is it not?

Phi. The very ſame: You cannot miſs. Come, add $\frac{b}{x}$, $\frac{c}{x}$, and $\frac{d}{x}$ together.

Tyr. Becauſe the Quantities have a common Denominator, I only add the Numerators, *viz.* $b + c + d$, under which I place the common Denominator, x, and their Sum is $\frac{b+c+d}{x}$.

Phi. Very right.

EXAMPLE 2.

Add $\frac{d}{x}$, $\frac{c}{d}$, *and* $\frac{f}{a}$ *together.*

Tyr.

Tyr. First $x \times d \times a = xda$ for a C. D. and $b \times d \times a = bda$ N. N. Again, $c \times x \times a = cxa$ N. N. And lastly, $f \times d \times x = fdx$ N. N. which I add as under:

$$\text{N. Numerators } \left.\begin{matrix} bda \\ cxa \\ fdx \end{matrix}\right\} \textit{add}$$

$$\textit{Ans. } \frac{bda + cxa + fdx}{xda}$$

Or if you set down all the new Numerators over their common Denominators, and abbreviate them by *Case* the 4*th*. you will see they are the same, and will be equivalent to the *Fraction* given. Thus, $\frac{bda}{xda} = \frac{b}{x}$, $\frac{cxa}{xda} = \frac{c}{d}$, and $\frac{fdx}{xda} = \frac{f}{a}$.

Now I will ask you one, if you please, with *mixt Numbers*.

Phi. With all my Heart.

EXAMPLE 3.

Tyr. *Add.* $4x\frac{b}{g}$ *and* $cd\frac{a}{f}$ *together*.

Phi. The Quantities not being a like, I add them only by the Sign $+$, and add the *Fractions* to them also by the same Sign as they stand. Thus, their Sum is $4x\frac{b}{g} + cd\frac{a}{f}$ Or otherwise thus, $4x + cd + \frac{bf + ag}{gf}$ Or, $\frac{4xg + b}{g} + \frac{cdf + a}{f}$

Tyr. If it were not too much Trouble *Philomathes*, I should be glad you would demonstrate this a little plainer to me.

Phi.

Phi. That I'll do three Ways.

1. As the Quantities are unlike, I only place them one after the other, (according to *Case* 3. *Dialogue* 5) and their Sum is $a\ 4\ x\frac{b}{g} + cd\frac{a}{f}$. Or,

2. By reducing the *mixt Numbers* to *improper Fractions*, their Sum is $\frac{4\ xg + b}{g} + \frac{cdf + a}{f}$. Or,

3. Reduce the *Fractions* $\frac{b}{g} \times \frac{a}{f}$ to a common Denominator, you will have $\frac{bf + ag}{gf}$ to which add the whole Quantities, and you will have $4\ x + cd + \frac{bf + ag}{gf}$.

But however, I will even do more than you desired, you shall see the *numerical* Proof. Let $x = 4$, then will $4\ x = 16$, and let the *Fraction* $\frac{b}{g}$ be $= \frac{1}{2}$. Make $c = 4$, and $d = 3$, then will $c\,d = 4 \times 3 = 12$, and let the *Fraction* $\frac{a}{f}$ be $= \frac{2}{3}$. Now at your Leisure add $16\frac{1}{2}$, to $12\frac{2}{3}$, you will have $\frac{175}{6} = 29\frac{1}{6}$ = the *improper Fraction* $\frac{4\ xg + b}{g} + \frac{cdf + a}{f}$ which is in Numbers $\frac{32 + 1}{2} + \frac{36 + 2}{3} = \frac{175}{6} = 29\frac{1}{6}$ as before Q. E. D. Does this appear plain to you?

Tyr. You lay me under the greatest Obligations, *Philomathes*; it appears quite easy to me indeed.

Phi. I am glad of it, then you are qualified for *Subtraction*, and there is no Occasion to dwell any longer upon *Addition*.

SECT.

SECT. III.

SUBTRACTION *of* ALGEBRAIC FRACTIONS.

Tyr. HOW do you perform *Subtraction?*

Phi. By one general Rule, *viz.* If the *Fractions* have a common Denominator, ſubtract the Numerators, by placing the Sign (—) before that which is to be ſubtracted, and place the Difference over the common Denominator; and if they have different Denominators, reduce them (by *Caſe* 5. *Sect.* 1. of this *Dialogue)* to a common Denominator, and then ſubtract the Numerators as in *Subtraction* of *Algebraic Integers.*

EXAMPLE 1.

From $\frac{a}{b}$ *take* $\frac{d}{b}$.

Anſ. $\frac{a-d}{b}$.

	Ex. 2.	Ex. 3.
From	$\frac{x+b}{f+g}$	$\frac{12x+c+b}{ad+g}$
Take	$\frac{3b-x}{f+g}$	$\frac{2x+b-c}{ad+g}$
Difference	$\frac{2x-2b}{f+g}$	$\frac{10x+2c}{ad+g}$

Tyr. I muſt confeſs I do not apprehend theſe two laſt Examples.

Phi.

Phi. You do well to ſay ſo, for they are not to be underſtood by every Learner at firſt Sight. Obſerve then, the Numerators of *Example* 1. are $x + b$, and $3b - x$. Now (as before directed) I change the Sign of the *Fraction* to be ſubtracted, and then it will be $- 3b + x$, which added (for this you muſt remember to do when the Signs are changed) to $x + b$, makes $2x - 2b$ for the Difference. And thus you muſt proceed with *Example* 3. always remembering that the *negative* and *affirmative* Sign before the ſame Quantity deſtroy each other, and you will find that $10x + 2c$ remains. The ſame is to be obſerved if the *Fractions* have not a common Denominator after they are once reduced to it. And thus much for *Subtraction*, provided you underſtand it.

Tyr. I thank you for this freſh Inſtruction; I am now pretty well grounded in this Rule I believe.

SECT. IV.

MULTIPLICATION *of* ALGEBRAIC FRACTIONS.

Phi. WELL, *Tyrunculus*, what think you of *Multiplication?*

Tyr. I think I can work it without ſhewing, if I am not miſtaken.

Phi. That is right, *Tyrunculus*, I love to ſee you bold and courageous in every new Undertaking: Come then,

Multiply $\frac{a}{b}$ *by* $\frac{c}{d}$.

Tyr.

Tyr. If I remember, I am only to multiply the Numerators together for a new Numerator, and the Denominators for a new Denominator. Thus, $a \times c = ac$, *and* $b \times d = bd$: *Anſ.* $\frac{ac}{bd}$

Phi. You are very right. Now multiply $\frac{4b + c}{g + a}$ by $\frac{3d}{9x}$

Tyr. Firſt, $\overline{4b + c} \times 3d = 12bd + 3cd$ for a N. N. and $\overline{g + a} \times 9x = 9xg + 9xa$ for a N. D. So will the *Anſwer* be $\frac{12bd + 3cd}{9xg + 9xa}$.

Phi. Very well done, *Tyrunculus*, indeed.

Tyr. Now, Sir, give me Leave to aſk you one?

EXAMPLE 3.

Multiply $\frac{12ax + b - 25x}{x}$ by $2b + 6x$.

Phi. Why, *Tyrunculus*, the Multiplier being a *whole Number*, I make a *Fraction of it*, and it will be

Mult. $\frac{12ax + b - 25x}{x}$ by $\frac{2b + 6x}{1}$.

Then multiplying the Numerators and Denominators together, I have

Anſ. $\frac{24abx + 2bb - 50bx + 72axx + 6bx - 150xx}{x}$. Or rather, it is $24ab - 44bx + 72ax - 150x + \frac{2bb}{x}$ *abbreviate.* Do you underſtand it?

Tyr. Yes, very well, except the abbreviated Anſwer.

Phi.

Phi. Pray look at *Case* the 4*th.* in *Reduction.* You see as I have x for the Denominator, I cast away or cancel x also in every Part of the Numerator; and as there is $-50\,bx+6\,bx$, this will be $-44\,bx$; then cancelling x, it is $-44\,b$, and x at last is set under the Quantity $2\,abb$ *Fraction-wise*, because x is not found in it.

Tyr. I heartily thank you, *Philomathes*, for this Information.

Phi. EXAMPLE 4.

Mult. $b\,\frac{c}{d}$

by a

$$\frac{dba+ca}{d}\ Ans.$$

EXAMPLE 5.

Mult. $4\,bx\,\frac{c}{d}$

by $\frac{5b}{9x}$

$$Ans.\ \frac{20bbxd+5cb}{9\,dx}$$

Reduce the *mixt* Numbers to *improper Fractions*, and proceed as before, you will have the Answers as above; which I beg you would try at your Leisure.

Tyr. I thank you, *Philomathes*; I will try it directly. Pray have you any Thing further to say of this Rule?

Phi. Nothing but this, that when it is required to multiply any *Fraction* by its Denominator; then cast away the Denominator, and the Numerator alone is the Answer. Thus, $\frac{d}{x}\times x$ gives d: For $\frac{d}{x}\times\frac{x}{1}=\frac{dx}{1x}$ or $\frac{dx}{x}=d$. So also, $\frac{9\,b+7\,dx}{4\,x+d}\times 4\,x+d$, gives $9\,b+7\,dx$ *Ans.* $=$ to the Numerator. This is evident; for suppose $\frac{3}{7}$ to be multiplied

plied by 7, it is $\frac{3}{7} \times \frac{7}{1} = \frac{21}{7} = 3$ the Numerator of the original *Fraction* $\frac{}{7}$, which in Fact is, only multiplying its Numerator by Unity or 1. And now, *Tyrunculus*, we will proceed to *Division.*

SECT. V.

DIVISION *of* ALGEBRAIC FRACTIONS.

Tyr. I Dare ſay I ſhall not be able to work *Diviſion* without ſhewing.

Phi. Poh! You are now going to be dead-hearted again, and without Cauſe; and I had much rather find you as bold as you were in *Multiplication.* Conſider, *Tyrunculus*, that every Learner may be compared to a young unexperienced Soldier; and though we will not call *Arithmetic* his Enemy, yet he has got many Skirmiſhes to go thro', and muſt not only *fight*, but that *valiantly* too, to overcome them; for a Field is ſeldom won by Cowardice: Beſides, *Tyrunculus*, I have hitherto furniſhed you with Weapons proper for ſuch Engagements as you have met with, and I ſhall take Care to provide you with others for every freſh Attack; and do you but learn to handle them well, and you need not fear but you will always overcome.

Tyr. You lay me under the higheſt Obligations to love and thank you, for being ſo careful of me. Pray then how is *Diviſion* performed?

Phi. The ſame as in *Vulgar Fractions. Multiply the Numerator of the Dividend unto the Denominator of the Diviſor for a new Numerator, and the Denominator of the Dividend into the Numerator of the Diviſor for a new Denominator.*

Ex. 1.	Ex. 2.	Ex. 3.
Divide $\frac{b}{g}$ by $\frac{d}{c}$	$\frac{14\ b}{g}$ by $\frac{2}{a}$	$\frac{4xx}{g}$ by $\frac{b}{4c}$
Anſ. $\frac{bc}{gd}$	$\frac{14\ ba}{2\ g} = \frac{7ba}{g}$	$\frac{16\ xxc}{bg}$

☞ *Tyr.* I underſtand it very well; but ſuppoſe the *Fractions* to have one and the ſame Denominator?

Phi. Then caſt away both the Denominators, and divide the Numerators only.

EXAMPLE. 4.

Divide $\frac{ba+bx+ca+cx}{5gg+d}$ by $\frac{b+c}{5gg+d}$. Then it will be thus, Divide $ba+bx+ca+cx$

$$
\begin{array}{l}
\text{by } b+c)\,ba+ca \qquad (a+x \text{ Anſ.} \\
\qquad\qquad \overline{} \\
\qquad\qquad 0 \quad bx+cx \\
\qquad\qquad\quad\;\; bx+cx \\
\qquad\qquad\quad\;\; \overline{} \\
\qquad\qquad\qquad\;\; 0
\end{array}
$$

Do you underſtand theſe Examples?

Tyr. Yes I do quite well.

Phi. Where then is the Difficulty you ſo much apprehended?

Tyr. I muſt confeſs that I was a little fearful juſt now.

Phi. I know it, I could ſee it in your Countenance.

Tyr. You have been ſo kind, that I muſt confeſs I cannot value you too much; nor can I repay you but with Thanks.

Phi.

Phi. I do it to ſerve you, as I obſerv'd, and if you receive Benefit by my Inſtructions, return the Thanks elſewhere. I only deſire you to be chearful and courageous, not timorous, for that will hinder you in your Purſuit.

Tyr. I will endeavour to follow your Advice in every Reſpect; but it is a dull Study for Learners for the firſt few Rules.

Phi. I own it is; but now you are paſt the worſt: You have in a great Meaſure drawn aſide the Maſk, and as ſoon as you are acquainted with the Rule of *Proportion*, and underſtand a little of *Equations*, (which you will ſoon do) it will then fall quite off, and you will with Pleaſure be raviſhed with the Beauty of its Face, and the Symmetry of its Parts.

Tyr. How long will it be before I come to the Rule of *Proportion?*

Phi. I ſhall ſhew you the Nature of it directly.

DIALOGUE VII.

SECT. I.

Of PROPORTION *in general.*

Tyr. WHAT do you mean by *Proportion?*

Phi. *Proportion* is the Relation, Reſpect, or Quality, that Numbers or Quantities bear to each other, by a certain *Ratio*, *Reaſon*, *Analogy*, or *Compariſon.*

Tyr. Is not *Ratio* and *Proportion* then all one and the ſame?

 Phi.

Phi. They are often used and spoken of as one and the same Thing, but there is a Difference; for strictly speaking, *Ratio* is not *Proportion* itself; but shews the Difference of Numbers, by comparing one with the other; so that by it are seen the *Reason* or *Analogy* that the *Antecedent* bears to the *Consequent*, by such and such Comparison, and in Course the *Proportion* that the Numbers bear to each other.

Tyr. What do you mean by *Antecedent* and *Consequent?*

Phi. In any two Numbers or Quantities, the first Term is called the *Antecedent*, the second the *Consequent*. Thus, 4, 8, *&c.* 4 is the *Antecedent*, and 8 the *Consequent*; the *Ratio* is 4, because 8 is 4 + 4, and the *Comparison* is 2, because 4 and 8 compared, one is twice the other. So also in any Series of Numbers, as 2, 6, 10, 14, 18, *&c.* 2 and 18 are the *Antecedent* and *Consequent*, and all the others between them are both *Antecedents* and *Consequents*.

Tyr. I understand you very well but pray how many Sorts of *Proportion* are there?

Phi. There is, 1 *Disjunct Proportion*, or *the Rule of* 3. *direct*. 2. *Arithmetical Proportion*, or *Progression*. 3. *Geometrical Proportion*, or *Progression*. 4. *Duplicate Proportion*. 5. *Triplicate Proportion*. 6. *Harmonical Proportion*. And, 7. *Contra-harmonical Proportion*, &c. But it will be sufficient for our present Purpose to speak of the first *Three* only, since the Knowledge of the others depend upon these.

SECT.

SECT. II.

Of DIRECT PROPORTION, *or the* RULE *of* 3.

Tyr. WHAT have I more to do with *Proportion*, ſince I can work the *Rule of* 3. very well?

Phi. That may be; and yet you may not rightly underſtand the Nature of it. A great many Perſons deceive themſelves in this, for though they can work a Queſtion according to the Order of the Rule itſelf, yet they are quite ignorant of the *Relation* or *Proportion* which one Number bears to another, notwithſtanding it is evidently known that one Half of the *Mathematics* depend upon it.

Tyr. I thought if I could but barely work the Rule it was enough, (till you ſhew'd me to the contrary in *Abbreviations*;) but ſince I ſee it is of ſuch excellent Uſe, I beg you would explain the Nature of it a little plainer to me.

Phi. I intend it; but before I give a Demonſtration, it will be quite neceſſary that you ſhould be well acquainted with the following Obſervations, and then the Demonſtration will appear quite plain to you: So that my Advice is, you would read them over again and again.

OBSERV. I.

Any three Numbers or Quantities being propounded, after you have found a fourth Number in Proportion, according to the Order of the *Rule of* 3. direct, then the Proof of ſuch Work is eaſily

difcovered; for if it be done right, the Proportion will always hold thus;

As the 1*ft* : 2*d* : : 3*d* : 4*th*.

Or, as the 1*ft* : 3*d* : : 2*d* : 4*th*. That is, the 1*ft* bears the fame Proportion to the 3*d*, as the 2*d* does to the 4*th*.

OBSERV. 2.

The Product of the 1*ft* and 4th is equal to the Product of the 2*d* and 3*d*. That is, the Product of the *Extremes* is equal to the Product of the *Means*: For the 3*d* divided by the 1*ft*, is equal to the 4*th* divided by the 2*d*, &c.

OBSERV. 3.

The 1*ft* is equal to the Product of the 2*d* and 3*d*, divided by the 4*th*.

OBSERV. 4.

The 2*d* is equal to the Product of the 1*ft* and 4*th*, divided by the 3*d*.

OBSERV. 5.

The 3*d* is equal to the Product of the 1*ft* and 4*th*, divided by the 2*d*.

OBSERV. 6.

The 4*th* is equal to the Product of the 2*d* and 3*d*, divided by the 1*ft*.

Tyr. I could not have thought there had been fuch Harmony in the Rule of *Proportion*. But pray explain thefe Obfervations to me by fome Demonftration?

Phi. I will, both by Quantities and Numbers. which, if you mind, you cannot but underftand it.

DE-

DEMONSTRATION.

Let the 4 Quantities x, b, c, and d, reprefent any 4 Numbers in direct Proportion, *viz.*
Let $x = 2$, $b = 4$, $c = 12$, and $d = 24$.

		Literally.		Numerically.	
	1	As $x:b::c:d$	i.e.	As $2 : 4 :: 12 : 24$	*Obf.* 1.
Or	2	As $x:c::b:d$		As $2 : 12 :: 4 : 24$	
For	3	$xd = cb$		$2 \times 24 = 4 \times 12$	*Obfervation* 2.
And	4	$\frac{b}{x} = \frac{d}{c}$		$4 \div 2 = 24 \div 12$	
Again	5	$\frac{x}{b} = \frac{c}{d}$		$\frac{2}{4} = \frac{12}{24}$	
And	6	$\frac{x}{c} = \frac{b}{d}$		$\frac{2}{12} = \frac{4}{24}$	
Laftly	7	$\frac{c}{x} = \frac{d}{b}$		$\frac{12}{2} = \frac{24}{4}$	

From the four laft Steps, each Term is found as follows:

Term					
1	1	$x = \frac{bc}{d}$	i. e.	$2 = \frac{48}{24}$	*Obf.* 3.
2	2	$b = \frac{xd}{c}$		$4 = \frac{48}{12}$	*Obf.* 4.
3	3	$c = \frac{xd}{b}$		$12 = \frac{48}{4}$	*Obf.* 5.
4	4	$d = \frac{bc}{x}$		$24 = \frac{48}{2}$	*Obf.* 6.

I fhould think, *Tyrunculus*, the *numerical* Work is fo plain, that you cannot help underftanding the *literal*, fince the Steps of one anfwer to the other.

Tyr.

Tyr. Nothing can be plainer indeed. But pray what do you mean by *Steps?* and what is their Uſe?

Phi. Thoſe are the Steps that ſtand in the Margin of the Work, numbered 1, 2, 3, *&c.* Their Uſe is to ſhew the gradual Proceeding of the Operation, that you go on *gradatim*, or by Degrees; that is, Step by Step.

Tyr. Have you any Thing further to ſay of *direct Proportion?*

Phi. I have nothing more to add but this, that when Quantities or Numbers are in a *direct Proportion*, they are alſo *Proportionals* by *Alteration*, *Inverſion*, *Diviſion*, *Converſion*, and *Compoſition*, &c. See *Euc.* 5. *Def.* 12, 13, &c.

SECT. III.

Of ARITHMETICAL PROPORTION.

Tyr. WHAT is *Arithmetical Proportion?*

Phi. Numbers or Quantities are ſaid to be in *Arithmetical Proportion*, or *Progreſſion*, when they differ from one another by a certain *Ratio*, or the like Reaſon, Thus, 2, 6, 10, 14, 18, 22, *&c.* are Numbers in *Arithmetical Progreſſion*, becauſe they differ from one another by the like Reaſon, *viz.* by 4, which Difference is called the *Ratio*. So 1, 19, 37, 55, *&c.* differ from each other by the *Ratio* 18, as you may perceive; for $1 + 18 = 19$, $19 + 18 = 37$, $37 + 18 = 55$, *&c.* From hence will follow this Obſervation.

OBSERV.

Observ. 1.

Any 3 Numbers or Quantities in *Arithmetical Proportion*, the Double of the *Mean* (or middle Number) is equal to thu Sum of the *Extremes*.

Numerical Demonſtration.

Let the 3 Numbers be 5, 13, and 21, whoſe *Ratio* or common Difference is 8; the Double of the *Mean* 13, is equal to the Sum of the *Extremes*, viz. 5 and 21.

Literal Demonſtration.

Let x be put for the firſt Term 5, and let e repreſent the *Ratio*, which is 8. Then will $x + e =$ 13 the *Mean*, and $x + 2e = 21$ the third Term.

Proof.

Mean.	Extremes.
Add $\left\{\begin{array}{l} x + e = 13 \\ x + e = 13 \end{array}\right.$	$\left.\begin{array}{l} x + 2e = 21 \\ x = 5 \end{array}\right\}$ Add

M. doubled $2x + 2e = 26 = 2x + 2e = 26$ Sum of Ex.

Tyr. This is mighty plainly demonſtrated indeed! But pray muſt I always put this Letter (e) to repreſent the *Ratio?*

Phi. This is at your Option; you may uſe any Letters, provided you put one for the Terms, and another for the *Ratio*, or Difference of the Terms.

Observ. 2.

Any 4 Numbers or Quantities in *Arithmetical Proportion*, either continued or diſcontinued and interrupted,

rupted, the Sum of the *Means* is equal to the Sum of the *Extremes*.

Numerical Demonſtration.

Let the 4 Numbers in *Arithmetical Progreſſion* be 4, 16, 28, and 40, whoſe *Ratio* is 12; then it is plain that 16 + 28 the *Means*, is equal to 4 + 40 the *Extremes*.

Litterally.

Let a repreſent the firſt Term 4, and put x for the *Ratio* 12; then will $a + x = 16$, one *Mean*, and $a + 2x = 28$, the other *Mean*, and $a + 3x = 40$, the laſt Term or *Extreme*.

PROOF.

Means.	Extremes.
$a + x = 16$	$a = 4$
$a + 2x = 28$	$a + 3x = 40$

Means $2a + 3x = 44 =$ *Extremes* $2a + 3x = 44$.

N. B. It would be the ſame if the Numbers had been diſcontinued, provided the Interruption be between the 2*d* and 3*d* Term. Thus, ſuppoſe the 4 Numbers were 4, 16, 124, 136; then $4 + 136 = 16 + 124 = 140$. For there is the ſame *Ratio* between the 3*d* and 4*th*, as there is between the 1*ſt* and 2*d*, *viz.* 12.

Tyr. I heartily thank you, kind *Philomathes*: Have you any Thing further to add upon this?

Phi. I am not willing to leave any Thing out that may be ſerviceable; but I think I have ſaid enough upon this Rule for your preſent Occaſion. However, it may be expected I ſhould teach you to work ſome

ſome Queſtions, or at leaſt give you ſome Rules to work them by.

Tyr. I think that Mr. *Ward* (in his *Arithmetic, Page* 76) ſpeaks of twenty Theorems belonging to this Rule; but he has given Examples only of two of them, and the other 18 I find in his *Algebra, Page* 186, but having no Rule for them, they are (I ſhould think) beyond the Reach of moſt Learners. I ſhould have liked he had given the Rule for finding them, though he had not done the Operation itſelf; becauſe by a plain Theorem, or Rule to work by, any aſſiduous Learner would know how to put Things in Practice that are not very difficult; but how ſhould he know when he has no Rule to go by nor any Tutor at Hand.

Phi. Had he given you the work of ſix Theorems with their Rules, you might with Eaſe have found out the reſt; as you will diſcover by the ſix following *Caſes*; the 2*d* and 5*th* of which will anſwer to his two in *Page* 74 of his Work.

CASE 1.

The Number of Places or Terms, and the Ratio or common Exceſs being given, to find the laſt Number.

Multiply the Number of Places leſs one, by the *Ratio* or common Exceſs; and to that Product add the firſt Number, and the Sum will be the laſt Number.

CASE 2.

The firſt and laſt Number (viz. *the Extreme) and the Number of Terms being given, to find the Aggregate or total Sum of all the Series.*

Add

Add the firſt and laſt Numbers together, and multiply the Sum by half the Number of Places, and you have the Total of all the Series added together. Or, in Caſe the Number of Places be odd, then add the firſt and laſt Numbers together, and multiply the Sum by the whole Number of Places, and divide that Product by 2, and you have the Aggregate or total Sum.

CASE 3.

The Extremes and Total given, to find the Number of Terms.

Add the *Extremes* together, and divide the Total by their Sum, and the Quotient will be equal to Half the Number of Places.

CASE 4.

The Total and Number of Terms given, to find the laſt Number.

Divide the Total by Half the Number of Places, or in Caſe the Terms be odd, divide double the Total by the Number of Terms, and the Quotient will be a Number; from which if you take the firſt Term, the Remainder will be the laſt Number.

CASE 5.

The Extremes and Number of Terms given, to find the Ratio or common Exceſs.

From the greater take the leſs *Extreme*, and the Remainder ſhall be a Dividend; then from the Number of Terms take Unity, *(viz.* 1.*)* and the Remainder ſhall be a Diviſor; and the Quotient riſing from them ſhall be the *Ratio*, or common Difference of the Terms.

CASE 6.

The Extremes and common Excefs given, to find the Number of Terms.

From the greater take the lefs *Extreme*, and divide the Remainder by the common Excefs; then to the Quotient add Unity, or 1, and that Sum will be equal to the Number of Places.

Tyr. Thefe Rules are very plain indeed, they need no Example.

Phi. Example and Precept are beft together, therefore I will give you an *Example* in *Cafe* the 2*d.* and *Cafe* the 5*th.* and you will, no Doubt, do the reft upon firft Trial.

EXAMPLE *of* CASE 5.

Let the Number of Places be 8, the *Extremes* 4 and 39, I demand the *Ratio?*

Firft, $39 - 4 = 35$ Dividend, then $8 - 1 = 7$ the Divifor, and $35 \div 7$ gives 5 the common Excefs. Proof 4, 9, 14, 19, 24, 29, 34, 39.

Or literally thus:

Let $x = 4$ lefs Extreme, and $e = 39$ the greater, and $b =$ the Number of Terms; then will $\frac{e - x}{b - 1} = 5$ the *Ratio* as above.

Tyr. I underftand it very well. Now give me one Example of *Cafe* the 2*d?*

Phi. I will.

EXAMPLE *of* CASE 2.

Let the Numbers be as above, *viz.* 4, 9, 14, 19, 24, 29, 34, 39. It is required to find the Aggregate or total Sum of all the Terms added together.

Firſt Number or *Extreme*	4	
laſt	39	
	43	Sum
This × ½ Number of Terms, *viz.*	4	
Total	172	

Or literally thus:

Let x be the firſt, and e the laſt Number, and let b repreſent Half the Number of Terms.

$$\text{Then } x + e = 43 \text{ as above}$$
$$b = 4$$

$$\text{Total } bx + be = 172 \text{ as above.}$$

So that from hence you ſee another Rule to find the Total, *viz.* Multiply the leſs *Extreme* and the greater ſeparately by Half the Number of Terms, and add their Products together, it will be the Sum of all the Series. And thus, x, e, and b, may repreſent the *Extremes* and Half the Terms, be they ever ſo many, which you are carefully to obſerve.

Tyr. I like this very well, and I am ſure it is far from being hard.

Phi. I ſhall leave you a Queſtion to try at your Leiſure to ſee if your Anſwer be like mine.

Three or 4 Men in Company were diſputing concerning the Diſtance, and the Time it would take to gather up Stones laid each a Yard aſſunder for one Mile

Mile in Length, and bringing each Stone back to the Place they began at. A silly bragging Jockey (who had present a good Horse) said he could ride further than contained to that in 3 Hours. A Sharper in Company taking Advantage of his Folly, said he would venture him 50 Guineas he did not ride his Horse so far in 3 Days; the Jockey unwarily consents; the Wager is staked, and he was to set out next Morning; but long before this he found it better to yield it lost than make Trial of such an Impossibility, it being 1549680 Yards = 880 Miles and a Half; which is upwards of 293 $\frac{1}{3}$ Miles a Day.

Tyr. Surprizing! I will try at it very shortly. Pray what comes next?

Phi. Geometrical Progression.

SECT. IV.

Of GEOMETRICAL PROPOSITION.

Tyr. WHAT is *Geometrical Proportion?*

Phi. Geometrical Proportion, or *Progression*, is when Numbers or Quantities differ from each other by like *Ratio* or Reason, as in *Arithmetical Progression*, only with this Difference, that in *Arithmetical Progression* the *Ratio* is the Effect of *Addition*, but in *this* of *Multiplication*, by having one common Multiplier.

Tyr. Please to explain this more clearly to me?

Phi. Observe then, 2, 4, 8, 16, 32, 64, ∻ &c. are Numbers in *Geometrical Proportion*, and differ by double Reason the one from the other, the common Multiplier being 2. They are every one you see the Double of the preceding Number. So also 4, 12,

36, 108, *&c.* differ by triple Reaſon, each Term being three Times its preceding one. And 1, 4, 16, 64, 256, *&c.* differ by quadruple Reaſon, *&c. &c. &c.*

Tyr. I underſtand you now perfectly well.

Phi. Then you are to obſerve as follows.

OBSERV. I.

Any three Numbers in *Geometrical Proportion*, the Product of the *Extremes* is equal to the Square of the *Mean*; that is, equal to the middle Term multiplied by or into itſelf.

Let the 3 Numbers be 4, 16, and 64. Here $4 \times 64 = 16 \times 16 = 256$, *&c.*

Literal Demonſtration.

Let x repreſent the firſt Term or *Extreme*, and let e be put for the *Ratio*, then will xe be the *Mean*, and xee the laſt Term, or other *Extreme*; then will $x \times xee$ be $=$ the Square of the *Mean* xe, viz. $xxee$.

PROOF.

	Extremes.	Mean.
	$x = 4$	$xe = 16$
	$xee = 64$	$xe = 16$
Product	$xxee = 256$	$xxee = 256$

Tyr. I underſtand the Example very well.

Phi. Once more then obſerve.

OBSERV.

OBSERV. 2.

Any four Numbers or Quantities in ∺, either continued or interrupted (provided the Interruption be between the 2*d* and 3*d* Term) the Product of the *Means* is equal to the Product of the *Extremes*.

EXAMPLE

Let the 4 Numbers be 5, 15, 26, and 78 interrupted; then $5 \times 78 = 15 \times 26 = 390$. It will be eaſy to prove the ſame literally as above.

OBSERV. 3.

The *Ratio* of any Series of Numbers in ∺ continued, is found only by dividing any of the Conſequents by it Antecedent, that is, dividing any Number by the preceding Number.

OBSERV. 4.

When ever ſo many Numbers or Quantities differ by double Reaſon, and it is required to find the laſt Number of all, the general Way of moſt Perſons is to double the 1*ſt*, 2*d*, 3*d*, 4*th*, &c. Number, and ſo continue to do till they have doubled as often as there are Terms given. But,

☞ There is a better Way when the Places are a great many, for you have no Occaſion to double but a few of the Terms, and then multiply that Number into itſelf, and the Product will be the Double of the Terms wanting one; which doubled, gives the next Term, &c. &c.

Tyr. This muſt be further explained to me, I do not apprehend it.

Phi. It is a little dark in Words only; but you'll underſtand it the Moment you ſee it done.

Suppoſe then a Series of Numbers in ∺ from 1 to 80 Places were given, which differ by double Reaſon, and it was required to find the laſt Number. Firſt double a few of them, ſuppoſing to the 5*th* Place, (which may be done by the Head only) then ſquare this Number, it ſhall give you the 9*th* Term; which doubled, gives you the 10*th* Term; this ſquared, gives the 19*th* Term, which multiplied by 2, gives the 20*th* Term; this ſquared gives the 39*th* Term; which into 2 gives the 40*th*; this into itſelf gives the 79*th*; and laſtly, this doubled gives the 80*th* or laſt Term, &c. &c. &c.

Tyr. You need not demonſtrate it any further; but how ſhall I find the Sum or Total of all the Series?

Phi. Very eaſily, by either of the following Methods.

Observ. 5.

To find the Sum of all the Series.

1. Multiply the laſt Term by the *Ratio*, or *common Exceſs*, and from the Product ſubtract the firſt Term; then divide the Remainder by the *Ratio* wanting 1, and it will give you the Sum of the Series. Or rather,

2. From the laſt Term take the firſt, and divide the Remainder by the *Ratio*, or *common Exceſs*, leſs Unity or 1; then multiply the Quotient by the *Ratio*, and to that Product add the firſt Number, and you will have the Sum of all the Series.

Tyr. I heartily thank you, kind *Philomathes*, for your Trouble.

Phi.

Phi. I ſhall not give you any Examples at large, but only ſhew you that the Increaſe of Numbers in ∸ is beyond the Belief or Conception of People in general. Thus, a Horſe having 8 Nails in each Shoe, and being bought or ſold at only 1 Farthing the firſt Nail, and double the Price for the next till you come to the 32*d* Nail, would amount to the Sum of £ 4473924 5*s.* 3*d.* $\frac{3}{4}$. And one Nail more would make it £ 8947848. 10*s.* 7*d.* $\frac{3}{4}$. Thus alſo, if a Farmer's Servant ſhould agree with his Maſter to ſerve him 20 Years for 1 Grain of Wheat only the firſt Year, and 10 the next, and ſo to have 10 Times the Number every Year he would have 11111111111111111111 Grains for his Service. Now allowing according to the Standard, that 7860 Grains make a Pint the Number of Buſhels will be 22605613425926 nearly; which at ſo ſmall a Price as half a Crown a Buſhel will amount to £ 2825701678240: And ſuppoſe a Ship to carry 1000 Loads for her Burthen it would take 1000 Times more ſuch Ships in Number than the whole World can furniſh. Which according to the foregoing Rules you may try at your Leiſure.

Tyr. Surprizing indeed! depend upon it I will try it for Curioſity ſake; but pray before you finiſh this Head give an Idea of *Harmonical Proportion.*

Phi. Harmonical or *Muſical Proportion* is when in 3 Numbers given, the Difference of the 1*ſt* and 2*d* is to the Difference of the 2*d* and 3*d*, as the 1*ſt* is to the 3*d*; or when in 4 Numbers given, the Difference of the 1*ſt* and 2*d* is to the Difference of the 3*d* and 4*th*, as the 1*ſt* is to the 4*th*. And,

Contra-harmonical Proportion is when in 3 Numbers given the Difference of the 1*ſt* and 2*d* is to the Difference of the 2*d* and 3*d* as the 3*d*, is to the 1*ſt*; and ſuch are 6, 10, and 12.

Tyr.

Tyr. I am oblig'd to you. Pray what is the next Thing we are to learn?

Phi. You are now come to *Equations*, and pray take the greateſt Care you poſſibly can, for the ſolving of *Algebraic* Queſtions depend upon the true Knowledge thereof.

Tyr. I will be diligent to obſerve what you ſay.

DIALOGUE VIII.

Of EQUATIONS.

SECT. I.

Of REDUCTION

Tyr. WHAT do you mean by an *Equation?*

Phi. An *Equation* is an exact *Equality*, or the mutual Agreement of two or more Things when compared together. Thus, when a Pound *Sterling* is compared with Shillings, it is found equal to 20, and a Crown compared with Groats is equal to 15 ſuch Pieces; therefore there can be no *Equation* where there are not two Things at leaſt, becauſe there can be no *Analogy* or *Compariſon:* And when there are two Numbers or Quantities, or more, to be compared with each other, you will always find this Sign or Character ($=$) placed between them.

Demonſtration.

Suppoſe x to repreſent a £. *Sterling*, and d 240 Pence its Equivalent, then it is evident that $x = d$.

Again,

Again, Suppofe g to reprefent 5 Shillings, and e 15 Groats, then will $g = e$. But fuppofe g to reprefent a Shilling only, and e one Groat only, then there muft be Numbers before the Quantities to form an *Equation*; for whereas before g was equal to e, now here it will be $g = 3e$, or $5g = 15e$; *viz.* 1 *s.* = 3 Groats, 5 *s.* = 15 Groats, &c.

Tyr. I underftand the Demonftration very well.

Phi. You are further to obferve, *Tyrunculus*, that in every *Equation* there are two Parts; that Part which ftands before the Sign is called the firft Part, and that after it the Second.

Example.

Suppofe $x = 4b + c$, then is x on the firft Part equal to 4 Times the Quantity reprefented by b on the fecond Part, together with the Quantity reprefented by c added to it.

Tyr. Pray how are *Equations* formed?

Phi. This is a Queftion that I cannot anfwer as yet to your Underftanding; but you may learn thus far, that when one or more Letters, reprefenting any known Quantity, are found on the fame Side of the *Equation* with other Quantities that reprefent unknown Quantities; then they muft be fo managed as to be brought on the other Side of the *Equation*; fo that one Side of the *Equation* muft be poffeffed by unknown, and the other by known Quantities, with the Sign of *Equality* between them; and thus will the unknown Quantity be difcovered: And this is call'd *Tranfpofition*. From hence will follow thefe *Axioms*, or felf-evident Principles, which I beg you would get by Heart, at leaft fo as to know their Ufe and Meaning.

Axiom

AXIOM 1.

If equal Numbers or Quantities be added to equal Numbers or Quantities, their Sum will ſtill be equal; that is, ſuppoſe a was $=4$, then by adding any Number or Quantity to each Side of the *Equation* (ſuppoſe 12) it will ſtill be equal; that is, $a+12 = 4+12, = 16$, *&c.*

AXIOM 2.

If equal Quantities or Numbers be ſubtracted from equal Quantities or Numbers, the Remainder will be equal. Thus, ſuppoſe $x = 12$, then by ſubtracting 8 from each Side, $x - 8 = 12, - 8 = 4$, *&c.*

AXIOM 3.

If equal Quantities be multiplied by equal Quantities, the Products will be ſtill equal. Thus, ſuppoſe $x = 8$, and I multiply each Side by any Quantity or Number, as 12, then will $12x = 96$. This is plain from the next *Axiom.*

AXIOM 4.

If equal Quantities be divided by equal Quantities, the Quotients will be equal. Suppoſe $12x = 96$, then dividing by 12, x will be equal to 8, as *per Axiom* 3.

AXIOM 5.

Thoſe Numbers or Quantities that are equal to one and the ſame Thing, are equal to one another; that is, ſuppoſe x, or $b - c$, or $5e + 6$, were either of them $=$ to 144, then are they alſo equal to each other.

You will ſee more of the Nature of theſe *Axioms* in the next *Section,* in treating of *Tranſpoſition.*

SECT.

SECT. II.

REDUCTION *by* ADDITION, *or, the Method of* TRANSPOSING *Numbers and Quantities.*

Tyr. WHAT do you mean by *Transposition?*

Phi. Transposition is the *transposing, altering, changing,* or *removing* any Thing from one Place to another. To *transpose* then any Number or Quantity, is only to remove it from one Side of *Equation,* and placing it on the other with the contrary Sign; and this answers to *Axiom* the *1st* and *2d.*

Tyr. If I remember, you use this Character (ϕ) for *Transposition:* Do you not?

Phi. Yes, I shall throughout the Work, and where-ever you meet with it read the Word *transposing.*

Tyr. Very well. Please to give me some Examples in *Addition?*

☞ *Phi.* I will; and pray remember, that *Addition* is nothing more than removing every *negative Quantity* to the contrary Side of the *Equation,* and making it *affirmative.*

EXAMPLE I.

Suppose $\left|\begin{array}{l} 1 \\ 2 \end{array}\right| \begin{array}{l} x - b\ c. \text{ Then } \phi - b \\ x = c + b \textit{ Ans.} \end{array}$

Or by *Axiom* 1 adding $+ b$ to each Side of the *Equation,* it will be

$\left|\begin{array}{l} 1 \\ 2 \\ 3 \end{array}\right| \begin{array}{l} x - b = c \\ \quad + b = + b \text{ it is} \\ x = c + b \text{ as before; because } - b \times b \end{array}$

on the first Side of the *Equation* destroy each other.

Ex-

EXAMPLE 2.

$$\text{Let}\ \left|\begin{array}{c|l} 1 & x - d - b = g. \quad \text{Then } \varphi - b \\ 2 & x - d = g + b. \quad \text{And } \varphi - d \\ 3 & x = g + d + b. \ \textit{Anf.} \end{array}\right.$$

Or by adding $+ d + b$ to each Side,

$$\left|\begin{array}{c|l} 1 & x - d - b = g \\ 2 & x + d + b = + d + b, \text{ it will be} \\ 3 & x = g + d + b \text{ as before, becaufe } - d \end{array}\right.$$

$- b$ and $+ b$ on the firft Side deftroy each other.

Thus you fee *Tranfpofition* agrees with *Axiom* 1.

Tyr. I perceive it does; but it is lefs Trouble to change the Signs, than it is to add equal Quantities on each Side.

Phi. It is; but ftill *Axioms* 1. fhews you the Reafon of it, which perhaps you might not have known elfe.

Tyr. Pleafe to give me another Example, and prove it by Numbers?

Phi. Obferve then.

EXAMPLE 3.

$$\text{Let}\ \left|\begin{array}{c|l} 1 & x - bc - d = aa. \quad \text{Then } \varphi - d \\ 2 & x - bc = aa + d. \quad \text{And } \varphi - bc \\ 3 & x = aa + d + bc \ \textit{Anf.} \end{array}\right.$$

Numerical Proof.

The Equation is $x - bc - d = aa$. Make $- bc = - 12$, $- d = - 8$, and $aa = 25$. Then will it be

$$\left|\begin{array}{c|l} 1 & x - 12 - 8 = 25. \quad \text{Then } \varphi - 8 \\ 2 & x - 12 = 25 + 8. \quad \text{And } \varphi - 12 \\ 3 & x = 25 + 8 + 12 = 45 = aa + d + bc \end{array}\right.$$

as before.

And

And thus you fee that Quantities may be reprefented by any Numbers at Pleafure, and the Value of the unknown Quantity x may eafily be difcovered.

Tyr. I like this very well indeed. Give me fome more Examples.

Phi. I will.

EXAMPLE 4.

Let	1	$5x - 8 = 24 - x$. Then $\varphi - 8$
	2	$5x = 24 + 8 - x$. And $\varphi - x$
	3	$5x + x$, that is, $6x = 24 + 8 = 32$ *Anf.*

EXAMPLE 5.

Let	1	$x - d - bc = aa - 12\,b$. Then $\varphi - bc$
	2	$x - d = aa - 12\,b + bc$. And $\varphi - d$
	3	$x = aa - 12\,b + bc + d$. Laftly, $\varphi - 12b$
	4	$x + 12\,b = aa + bc + d$. *Anf.*

Do you underftand it?

Tyr. Yes, very plainly.

Phi. Then we will proceed to *Subtraction*, in which I fhall give you the fame Sort of Examples as in Addition, that you may fee the Nature of both the better.

SECT. III.

REDUCTION *by* SUBTRACTION.

Tyr. I Underftand *Addition* very well, and apprehend *Subtraction* to be only the Reverfe of it.

Phi. You are right, for here you have Nothing to do but to *tranfpofe* the *affirmative* Quantities or Numbers to the other Side of the *Equation*, and place the *negative* Sign before them.

EXAMPLE 1.

Let
| 1 | $x + b = c$. Then $\varphi + b$
| 2 | $x = c - b$ *Anf.* See *Ex.* 1. *Addition.*

Or by *Axiom* 2, fubtracting $- b$ from each Side, it will be the fame. Thus,

| 1 | $x + b = c$. Then fubtracting
| 2 | $- b = - b$, it is
| 3 | $x = c - b$ as before; for $+ b - b$ deftroy each other on the firft Side.

EXAMPLE 2.

Let
| 1 | $x + d + b = g$. Then $\varphi + b$
| 2 | $x + d = g - b$. And $\varphi + d$
| 3 | $x = g - b - d$ *Anf.*

Or by fubtracting $- d - b$ from each Side.

| 1 | $x + d + b = g$. Then
| 2 | $- d - b = - d - b$
| 3 | $x = g - b - d$, as above; becaufe $+ d + b - d - b$ on the firft Side deftroy each other.

EXAMPLE 3.

Let
| 1 | $x + bc + d = aa$. Then $\varphi + d$
| 2 | $x + bc = aa - d$. And $\varphi + bc$
| 3 | $x = aa - d - bc$ *Anf.*

Numerical Proof.

The *Equation* is, $x + bc + d = aa$. Now let $+ bc = 12$, $d = 8$, and $aa = 25$. Then it will be

| 1 | $x + 12 + 8 = 25$. Then φ 8
| 2 | $x + 12 = 25 - 8$. And φ 12
| 3 | $x = 25 - 8 - 12 = 5$ *Anf.* $= aa - d - bc$, as above.

Pleafe to compare this with *Example* 3 in *Addition*, you will fee the Quantities are the fame, but the Difference

Difference of the Value of x is 40 leſs here than it is there, becauſe you ſee that what is *affirmative* there is *negative* here. And indeed I am of Opinion, that the comparing of them with each other will ſhew you more the Nature of each, than many Examples whoſe Steps are not alike, and confuſedly demonſtrated.

Tyr. Indeed I think it almoſt impoſſible not to underſtand it, ſo plain as you have done it.

Phi. Here follows then

EXAMPLE. 4.

Let	1	$5x + 8 = 24 + x$. Then $\varphi + 8$
	2	$5x = 24 - 8 + x$
	3	$5x - x$, that is, $4x = 24 - 8 = 16$.

See *Example* 4. in *Addition.*

EXAMPLE 5.

Let	1	$x + d + bc = aa + 12b$. Then $\varphi\ bc$
	2	$x + d = aa + 12b - bc$. And d
	3	$x = aa + 12b - bc - d$. Laſtly, $\varphi\ 12b$
	4	$x - 12b = aa - bc - d$ *Anſ.* Compare

this with *Example* 5. in *Addition.*

Tyr. I ſee the Nature of both plainly. Have you any Thing further to add?

Phi. It may not be amiſs to give you an Example to exerciſe you in both.

An EXAMPLE *in both* RULES

EXAMPLE 5.

Let	1	$4x - g + b + c - d = 142$. By $\varphi - g$,
	2	$4x + b + c - d = 142 + g$. By $\varphi\ b$,
	3	$4x + c - d = 142 + g - b$. Then $\varphi\ c$,
	4	$4x - d = 142 + g - b - c$. And $\varphi - d$,
	5	$4x = 142 + g - b - c + d$ *Anſ.*

Numerical Proof.

Let $-g = -8$, $+b = 12$, $+c = 6$ and $-d = -16$, What will x be?

Then,

1	$4x-8+12+6-16=142$. By $\phi -8$,
2	$4x+12+6-16=142+8$. By ϕ 12,
3	$4x+6-16=142+8-12$. By ϕ 6,
4	$4x-16=142+8-12-6$. And $\phi -16$,
5	$4x=142+8-12-6+16$ *Anf.* as above. That is,
6	$4x=166-18$. That is.
7	$4x = 148$. Therefore by dividing 148 by 4,
8	$x = \frac{148}{4} = 37$ *Anf.*

☞ *Tyr.* I like this *numerical* Proof very much, it is fo plain, and the firft five Steps agree fo with the *literal*, that there needs no more Examples of this Sort.

☞ *Phi.* I have proceeded indeed but one Step and one Quantity at a Time, becaufe you might fee the gradual Order of the Work; but you may as well *tranfpofe* them all at one Stroke, for it is only ufing the contrary Sign you know; however, this is left to your Liberty and Practice.

Tyr. I underftand you very well. Pray what comes next?

Phi. You are now come to *Multiplication*, where you will begin to fee the Beauty of *Equations*.

SECT.

SECT. IV.

REDUCTION *by* MULTIPLICATION.

Tyr. HOW is *Multiplication* of *Equations* performed?

Phi. Multiplication is performed as follows:

1. When there is an *Equation* between two *Fractions* having a common Denominator, then caſt away or cancel the common Denominator, and the Numerators will be equal to each other.

EXAMPLE 1.

Let	1	$\frac{x}{9} = \frac{ab}{9}$. Then
	2	$x = ab$ *Anſ.*

2. Or if the *Fractions* have not a common Denominator reduce them to one, after which expunge the common Denominator, and the new Numerators will be equal.

EXAMPLE 2.

Let	1	$\frac{x}{5} = \frac{6}{10}$. Theſe reduced to a C. D.
	2	$\frac{10x}{50} = \frac{30}{50}$, that is,
	3	$10x = 30$ *Anſ.*

3. Or if there be but one *Fraction*, and that be made equal to any whole Number or Quantity; then only multiply the whole Quantity by the Denominator of the *Fraction*, and that Product ſhall be equal to the Numerator.

EXAMPLE 3.

Let	1	$x = \frac{24a}{12}$, then
	2	$12x = 24a$ *Anſ.*

4. Or, to prevent the Trouble of reducing the *Fractions* to a common Denominator, multiply the Numerator of the ſecond *Fraction* by the Denominator of the firſt *Fraction*, and place the Product for a new Numerator over the ſecond *Fraction*, ſo will the Numerator of the firſt *Fraction* be equal to it in the Second Step. Then multiply the Numerator of the firſt *Fraction* by the Denominator of the ſecond *Fraction*, ſo will this Product be equal to the ſaid new Numerator, and the *Equation* will be cleared from *Fractions*, and the unknown Quantity diſcovered.

EXAMPLE 4.

Let	1	$\frac{x}{12} = \frac{108}{4}$. Then 108×12
	2	$x = \frac{1296}{4}$. Then $x \times 4$
	3	$4x = 1296$. Then muſt
	4	$x = \frac{1296}{4} = 324$ *Anſ.*

Tyr. I like this Way beſt I muſt confeſs.

Phi. When the Numbers or Quantities are many, it is leſs Trouble indeed to do it this Way. Now perhaps you will be diverted with the Proof.

Tyr. I ſhould be glad to ſee the Reaſon indeed why $\frac{x}{12}$ is equal to $\frac{108}{4}$.

Phi.

Phi. That you ſhall directly, and in a few Words. Firſt, x is proved to be equal to 324 as above; then $\frac{324}{12} = \frac{108}{4} = 27$. So that $\frac{x}{12} = \frac{108}{4}$ Q. E. D.

Tyr. Very pretty indeed! But pray give me Leave to ſet you a Queſtion, and to deſire you to do the Work at large: I aſſure you it will be of great Service to me, and I ſhall need no more Examples of this Sort?

Phi. You know, or may know, I am always ready to ſerve you. Pray propoſe the Thing?

EXAMPLE 5.

Tyr. *Suppoſe* $\frac{3x}{4} \times 12 = \frac{2x}{3} + 14$, *what then is* x *equal to?*

Phi. A very pretty uſeful Queſtion, and I will do it ſo plain, that I believe you will be ſatisfied with the Manner of it, becauſe I ſhall demonſtrate it as a ſtanding Rule for all ſuch Examples. Let $\frac{3x}{4} + 12 = \frac{2x}{3} + 14$.

DEMONSTRATION.

Firſt, I multiply the whole Number 12, and the Numerator of the ſecond *Fraction*, (*viz.* $\frac{2x}{3}$) and the whole Number 14 into the Denominator of the firſt *Fraction*, *viz.* 4, and the Products are 48, $8x$ and 56: But whereas the Sign of Equality ($=$) falls between the whole Number 12 and the *Fraction* $\frac{2x}{3}$, I ſtill keep it always in the ſame Place, till I have done multiplying the Whole; therefore it will be $3x$ +

$+ 48 = 8x + 56$, under which I put the Denominator of the second *Fraction*, and so is the first Side of the *Equation* cleared of *Fractions*, and will stand in the second Step, thus, $3x + 48 = \frac{8x}{3} + 56$. Then to clear the second Side from *Fractions*, I now multiply every Member of the second Step into the Denominator of the second *Fraction*, *viz.* 3, (except it be its new Numerator $8x$) and then it will be in the third Step $9x + 144 = 8x + 168$, and thus is the whole *Equation* freed from *Fractions*. Now transposing $8x$ and $+ 144$, I have in the fourth Step $9x - 8x = 168 - 144$; that is, $x = 168 - 144 = 24$, the Value of x required. (See the Work at large as follows, and compare it with what is above.)

The Operation of EXAMPLE 5.

Let	1	$\frac{3x}{4} + = \frac{2x}{3} + 14$. Then $\times$ 4
	2	$3x + 48 = \frac{8x}{3} + 56$. Which $\times$ 3, the Denominator of the second *Fraction* is
	3	$9x + 144 = 8x + 168$. Then $\phi\, 8x + 144$,
	4	$9x - 8x = 168 - 144$. That is,
	5	$x = 24$. *Anf.*

PROOF.

To prove that $\frac{3x}{4} + 12 = \frac{2x}{3} + 14$.

First $x = 24$ as above, then must $\frac{3x}{4} + 12$, that is, $\frac{72}{4} + 12 = 30$; and so also $\frac{2x}{3}$ that is $\frac{48}{3} + 14 = 30$. Q. E. D. *

Tyr.

* *N. B.* This is called *Synthetical Demonstration*, or *Composition*, and you may see this *Equation* turned into a *Problem*, and solved *Algebraically*, *Dialogue* 10, *Problem* 29.

Tyr. Nothing can be plainer, nor more eafy to be underftood.

Phi. If due Attention be given, as you obferve, it is eafy enough; I fhall therefore leave one more Example with you, and haften to *Divifion*.

EXAMPLE 6.

Let	1	$\frac{4x}{9} + 16 = \frac{3x}{6} - 18$. Firft $\times$ 9
	2	$4x + 144 = \frac{27x}{6} - 162$. This $\times$ 6,
	3	$24x + 864 = 27x - 972$. Then ¢ $24x$ and -972,
	4	$864 + 972 = 27x - 24x$. That is,
	5	$1836 = 3x$. Therefore
	6	$x = \frac{1836}{3} = 612$ *Anf.*

Again,

Suppofe that $36 + \frac{6x}{8} = 72$, then will x be found to be $= 36$.

☞ But here, *Tyrunculus*, you muft obferve, that if at any Time the *Square* or *Cube* of any unknown Quantity fhould in the laft Step of any *Equation*, be found to be equal to fuch Number or Quantity, then muft you extract the *Square* or *Cube Root* of fuch Numbers, and you will then have the Value of the unknown Quantity itfelf. Thus, fuppofe xx fhould at laft fall out to be equal to 81, then is $x = 9$ the *Square Root* thereof; and if $xxx = 64$, then will $x = 4$ its *Cube Root*.

Ex-

EXAMPLE 7.

Let	1	$\frac{xx}{4} = 44 + 5.$ Then
	2	$xx = 176 + 20$, that is,
	3	$xx = 196$. Therefore,
	4	$x = \sqrt{196} = 14$ *Anſ.*

EXAMPLE 8.

Let	1	$\frac{xx}{b} + c + f = \frac{dg}{x}$
	2	$xx + bc + bf = \frac{bdg}{x}$
	3	$xxx + bcx + bfx = bdg$
	4	$xxx = bdg - bcx - bfx.$
	5	$x = \sqrt[3]{bdg - bcx - bfx}.$

Tyr. I need no more Examples, what you have ſhewn me already is ſufficient.

Phi. You may indeed reduce any ſimple *Equation* by what you have ſeen, therefore we will proceed to *Diviſion.*

SECT. V.

REDUCTION *by* DIVISION.

Tyr. PRAY how is *Diviſion* of *Equations* performed?

Phi. When any Quantity or Quantities that are alike, poſſeſs both Sides of the *Equation*, divide each Side

Side by the ſaid Quantity, (which is the ſame as to reduce it to its loweſt Terms) and then will one Side be ſtill equal to the other; and if there be *Fractions*, clear the *Equation* of them, by multiplying all the Parts by the Denominators of the *Fraction*, as in *Multiplication*.

EXAMPLE. 1.

Let	1	$xx = 16x + 12x$, then $\div$ by x,
	2	$x = 16 + 12 = 28$, by *Axiom* 4.

EXAMPLE 2.

Let	1	$xxc + bc\,x + dcx = cx + ffx$. Then $\div c$
	2	$xx + bx + dx = x + ffx$. Then $\div x$,
	3	$x + b + d = 1 + ff$ *Anſ.*

EXAMPLE 3.

Let	1	$bx - cx = gg$. Then $\div \overline{b-c}$,
	2	$x = \frac{gg}{b-c}$ *Anſ.*

Tyr. I do not rightly apprehend this laſt Example. Can you demonſtrate it by Numbers?

Phi. Yes to be ſure, and will. Let $b = 8 - c = -4$, and $g = 12$, then $gg = 144$.

Numerical Proof of EXAMPLE 3.

Let	1	$8x - 4x = 144$, then $\div$ by $8 - 4$
	2	$x = \frac{144}{8-4} = \frac{gg}{b-c} = 36$ *Anſ.*

Tyr.

Tyr. I heartily thank you, *Philomathes*, and am mightily pleaſed with it. But if there be *Fractions*, what do you ſay I am to do as I did in *Multiplication?*

Phi. Yes, after having firſt abbreviated the Numerators, or dividing them by like Quantities or Numbers, (for the Denominators are never divided) you proceed then to multiply croſs-ways, as in *Diviſion* of *Fractions*, till you diſcover the unknown Quantity.

Tyr. Pleaſe to give me an Example?

Phi. I will.

EXAMPLE 4.

Let	1	$\frac{84x}{x-4} = \frac{70x}{x-6}$. Then ÷ Numerators by x, you have
	2	$\frac{84}{x-4} = \frac{70}{x-6}$. Then $x - 4 \times 70$
	3	$84 = \frac{70x-280}{x-6}$. Then $84 \times x - 6$,
	4	$84x - 504 = 70x - 280$. Then φ $70x$ and -504,
	5	$84x - 70x = 504 - 280 = 224$; that is,
	6	$14x = 224$. Therefore,
	7	$x = \frac{224}{14} = 16$ *Anſ.*

Tyr. And can you prove this laſt Example *ſynthetically*, as you did in *Multiplication?*

Phi. Yes, moſt certainly, if the Work be done right.

A ſynthetical Proof of EXAMPLE 4.

You ſee that x is $= 16$, then $\frac{84x}{x-4}$, that is, $\frac{84 \times 16}{16 - 4}$ $= 112$, the firſt Side of the *Equation*. Again, $\frac{70x}{x-6}$, that

that is, $\frac{70 \times 16}{16 - 6} = 112$. Confequently therefore, $\frac{84x}{x-4} = \frac{70\ x}{x-6}$ Q. E. D.

Tyr. Not one Thing that you have demonftrated pleafes me better, nor gives me greater Satisfaction.

Phi. I will give you an Example or two more.

EXAMPLE 5.

Let	1	$ddxx + ddbxx - ddx = ddfx + ddffx$. Then $\div dd$
	2	$xx + bxx - x = fx + ffx$. Then $\div x$
	3	$x + bx - 1 = f + ff$.

☞ Here indeed the known Quantities and the unknown Quantity (which we fuppofe is reprefented by x) both poffefs the firft Side of the *Equation*, in Order therefore to let the unknown Quantity x poffefs one Side by itfelf, do thus: Let the whole *Equation* be divided by the known Quantity or Quantities, and then will the unknown Quantity x be equal to the Quotient of fuch Divifion. As,

EXAMPLE 6.

Suppofe in any *Equation* it fhould fo fall out, that $xd - xb = c + g$, what is x equal to?

Let	1	$xd - xb = c + g$. Then $\div d - b$
	2	$x = \frac{c+g}{d-b}$ *Anf.*

EXAMPLE 7.

☞ Again, Suppofe in trying to difcover the unknown Quantity, all the Quantities happen to fall together, fo as there is no Sign of *Equality* between,

then,

then, in Order to form an *Equation*, make ſuch Quantities the firſt Side, and put a Cypher on the ſecond Side of the *Equation*, ſo will the unknown Quantity be diſcovered. Thus,

Suppoſe	1	$12x - 312$, then
	2	$12x - 312 = 0$; that is,
	3	$12x = 312$. Therefore,
	4	$x = \frac{312}{12} = 26$.

N. B. See the 9th and 10th Steps of *Problems* 18. *Dialogue* 10.

EXAMPLE 8.

Let	1	$5xxd + 8xx = xbc + 4b$, then $\div x$
	2	$5xd + 8x = bc + 4b$, then $\div 5d + 8$
	3	$x = \frac{bc + 4b}{5d + 8}$ *Anſ.*

SECT. VI.

How to convert or turn EQUATIONS *into* ANALOGIES, *and the contrary.*

Tyr. I Imagine that this *Section* depends upon a true Knowledge of the Nature of *Proportion*; does it not?

Phi. Moſt certainly, and therefore from what has been laid down in *Dial* 7. *Sect.* 2. it will be eaſy to convert any *Equation* into an *Analogy*, or right *Proportion*; and eſpecially ſince I ſhall take ſome of the ſame *Equations*, and refer you back to the former Work, to confirm you the better in what you are doing.

OBSERV.

OBSERV. I.

When any *Equation* (not having *Fractions*) is given to be converted into an Analogy, then it will be, as any of the *Quantities* or *Factors* on one Side are to any *other* on the other Side; ſo will the remaining *Quantity* or *Quantities* on the ſame Side, be to the remaining *ones* on the other Side, and *vice verſa.*

EXAMPLE I.

Let the *Equation* be $xd = cb$. *See the third Step in the Demonſtration.* Dial. 7. Sect. 2.

Let	1	$xd = cb$, then
	2	As $x : c :: d : b$. Or
	3	As $x : b :: c : d$. &c. For the 1 Term
	4	$x \times$ (the 4) $d =$ (2) $c \times$ (3) b. Conſequently,
	5	$xd = cb$.

Tyr. I perceive then, this is but a common Proof to *Proportion.*

Phi. Nothing more; for if you compare this *Example* with the ſix *Obſervations* laid down in *Dial.* 7. *Sect.* 2. you may make a great many more Steps of it than I have done.

Tyr. I ſee plainly the Manner of turning *Equations* into *Analogies* when both the Sides are whole Quantities; but ſuppoſe one Side be a *whole Number* or *Quantity*, and the other a *Fraction?*

Phi. Then you are to proceed as follows.

OBSERV. 2.

When any *whole Number*, or Quantity in an *Equation* is made equal to a *Fraction*, whoſe Nu-

merator has two Quantities and the Denominator but one; then break the Numerator into two ſuch Parts, which multiplied together, will produce the ſame, and make thoſe Parts the *Means*; then make the whole Quantity, and the Denominator of the *Fraction* the *Extremes*. Or in other Words, make the whole Quantity the firſt Term in the *Rule of Three*, the Denominator of *Fraction* the fourth, and the Numerator divided into two Parts as before directed, make the ſecond and third Term.

EXAMPLE 2.

Let	1	$x = \frac{bc}{d}$. Then
	2	$x : b :: c : d$. Or
	3	$x : c :: b : d$. That is,
	4	$xd = bc$. See *Demonſtration*, *Dial.* 7. *Sect.* 2.

Tyr. I underſtand it very well, but ſuppoſe both be *Fractions*, how then?

Phi. Certainly you forget *Tyrunculus*. Pray turn back to the fourth Step of *Dialogue* 7. *Sect.* 2. for I ſhall give you the ſame *Example*. Or if you remember what I told you in *Abbreviations*, you will find the *Analogy* will hold as follows.

OBSERV. 3.

As one Denominator is to the other, ſo will one Numerator be to the other; or as one Denominator is to its own Numerator, ſo is the other Denominator to its Numerator, &c. &c.

Ex-

EXAMPLE 3.

Let	1	$\frac{b}{x} = \frac{d}{c}$. Then as
	2	$x : c :: b : d$. Or as
	3	$x : b :: c : d$, *&c. &c.*

Tyr. I am obliged to you: Have you Nothing more to add?

Phi. I will give you one *Example* by Way of Exercise.

EXAMPLE 4.

Let	1	$xb + xd = bd$. Then as
	2	$x : b :: d : b + d$. For
	3	$x \times \overline{b+d} = xb + xd$, the first Side. And
	4	$b \times d = bd$, the second Side. Or, as
	5	$b : x :: b + d : d$. Or, by adding xd to each Side,
	6	$xb + 2xd = bd + xd$. Then, as
	7	$x : b + x :: d : b + 2d$. Or, taking xd from each Side,
	8	$xb = bd - xd$. Then, as
	9	$x : d :: b - x : b$, *&c. &c.* consequently
	10	$xb + xd = bd$.

Tyr. Then I perceive by the third and fourth Steps, that if one Side of an *Equation* can be divided into two Parts, so as to become *Extremes* (which being multiplied together, will be equal to the Side before it was divided) the other Side being divided in the same Manner, will be the *Means*; will it not?

Phi. Yes, your Notion is right; and I am glad to find you so perfect in what you have done: Therefore I shall bid you *adieu*, and leave you to consider

upon thoſe *Examples*, which you think yourſelf leaſt acquainted with; and when Opportunity ſuits, I ſhall be glad to ſee you and your Friend *Novitius*, and then we will put theſe *Examples* in Practice by ſome *Algebraic Problems*.

Tyr. Sir, I am obliged to you; and I dare ſay *Novitius* will be as proud of the *Invitation* —— But let me beg of you to ſtay a little longer.

Phi. Not now, *Tyrunculus*, I think I have made you a long Viſit; beſides, Night comes on a-pace, and I chooſe to go.

Tyr. Sir, if you are determined to go, I heartily wiſh you a good Night, and humbly thank you for your Company, and I intend to do myſelf the Pleaſure of waiting upon you very ſhortly.

Phi. When you think proper *Tyrunculus*.

CHAP.

CHAP III.

DIALOGUE IX.

SECT. I.

Between PHILOMATHES *and* TYRUNCULUS, *concerning the Nature of* Algebraic Problems, *and how to prepare them for a* Solution.

Tyrunculus *returning the Visit to* Philomathes.

Tyr. P*Hilomathes*, your humble Servant, how do you do?

Phi. Thank you, *Tyrunculus*, I am pretty well, and am glad to see you so.

Tyr. You remember I said it should not be long before I would call again to see you; but, perhaps, I am not come at a suitable Time.

Phi. You could not have hit upon it better, *Tyrunculus*, it suits me quite well, and I was but just before thinking of, and wishing for you. — Come, pray sit down, — But where is your Friend *Novitius*, I expected you both together?

Tyr,

Tyr. We are obliged to you, Sir, and I aſked him to come, as you deſired, and he promiſed to follow me.

Phi. Well, *Tyrunculus*, in the mean Time let me know how you go on, and what Improvement you have made ſince I ſaw you laſt.

Tyr. I am afraid it will not bear too cloſe an Examination: However, that very Night you left me, I looked over the Chief of what you have ſhewn me, and find myſelf much more perfect in it.

Phi. You have done well, it is all I required, and you will be the better able to underſtand the following *Problems*.

Tyr. I muſt confeſs I do not care how ſoon I begin to try a few Queſtions, or at leaſt ſee them wrought, for you muſt know I am in a Hurry.

Phi. You ſhall preſently; but pray be not ſo over haſty; *fair* and *ſoftly*, you know, go the furtheſt; and I have Something to premiſe firſt of all, that will be of Service, and help you forward in the Work.

Tyr. Pray what is that?

Phi. It would be requiſite that you ſhould be acquainted with the following Obſervations.

OBSERV. I.

When any Queſtion is given to be anſwered in an *Algebraic* Manner, *firſt*, For the Anſwer or Number ſought, put x: Then proceed according to the Tenor of the Queſtion, to *add*, *ſubtract*, *multiply*, or *divide*, until you have formed an *Equation*, which if it has *Fractions* muſt be cleared according to the Rules laid down in *Multiplication*. *Sect.* 4. *Dial* 8: This done, proceed to *tranſpoſe* according to the Order of *Addition* and *Subtraction* of *Equations*, and you will (by keeping x on the firſt Side of the *Equation*)

tion) have it equal at laſt to ſome known *Quantity* or *Quantities*, by which alſo x will be of Courſe known, and its Value diſcovered.

OBSERV. 2.

Though it be cuſtomary to uſe x for the unknown Quantity, yet you may make Uſe of any other Letter at Pleaſure. Some *Analiſts* uſe Vowels to repreſent *unknown*, and Conſonants *known* Quantities; but others uſe them as their own Fancy and Inclination direct: But ſtill you are to obſerve, the Letter *(o)* is never uſed to expreſs a Quantity, (though indeed the Anſwer would be the ſame with this as with any other Letter,) and there ſeems to be a Reaſon for it, ſince it is but a Cypher at beſt without Integers; and therefore, ſince Nothing cannot be Something, by Reaſon of its Want or Deficiency, it would be abſurd to put it to repreſent any Number or Quantity; though, as I obſerved before, it is ſometimes uſed to form an *Equation*, See *Problem* 18. *Step.* 10.

OBSERV. 3.

If to the Sum of any two Numbers you add their Difference, and divide the Whole by 2, the Quotient will be the greater Number. Or if you add the Numbers and their Difference together, and divide by 2, you have the greater Number.

OBSERV. 4.

If from the greater Number you take the Difference of the ſaid Numbers, the Remainder will be the leſs Number. Or, if you add any two Numbers and their Difference together, and divide the Sum

Sum by 2, and then ſubtract the Difference of the ſaid Numbers from the Quotient, the Remainder will be the leſs Number.

☞ OBSERV. 5.

When any *Fraction* is given to be divided into two, three, or more Parts, then divide the Numerator, if you can, into ſuch Parts, and let the Denominator remain as it was; and in Caſe you cannot divide the Numerator into the Parts required, multiply the Denominator into ſuch Parts as are requir'd, and let the Numerator remain as it was; ſo is the *Fraction* truly divided into ſuch Parts as really as if it had been performed by *Diviſion*, which is ſometimes very difficult. Thus, ſuppoſe I was to divide $\frac{6}{x}$ into three Parts, I divide 6 by 3, and it is 2; ſo is $\frac{2}{x}$ the $\frac{1}{3}$ of $\frac{6}{x}$. But ſuppoſe it were $\frac{x}{6}$ to be divided into 3 Parts, as I cannot well divide x by 3, therefore I multiply the Denominator 6 by 3, and it is 18; ſo is $\frac{x}{18}$ $\frac{1}{3}$ of $\frac{x}{6}$. This I have demonſtrated, becauſe you ſhall ſeldom meet with it in any Authors, although it is of infinite Service in *Algebra*. I beg therefore you would remember it in particular.

OBSERV. 6.

When any two Numbers are given, and you would expreſs them *literally*, (we will ſuppoſe you put x for the greater, and e for the leſs Number) then will the following Steps be of Service, becauſe they will help you to underſtand the Nature of a Queſtion, and the ſooner to do the ſame, as being a pro-

per and neceſſary Exerciſe of the foregoing Rules, by teaching you how to expreſs them in their true Order. Thus,

1 Suppoſe the greater be x,
2 And the leſs Number e,
3 Then will their Sum be $x + e$,
4 Their Difference $x - e$,
5 Their Product $x \times e$, *viz.* xe,
6 Their Quotient of the greater $\div$ by the leſs is $\frac{x}{e}$,
7 The Quotient of the leſs by the greater is $\frac{e}{x}$,
8 The Order of Proportion, as $x : e :: e : \frac{ee}{x}$.
9 Or, by putting the leſs firſt, as $e : x :: x : \frac{xx}{e}$.
10 The Square of the greater xx,
11 The Square of the leſs ee,
12 The Sum of their Squares $xx + ee$
13 The Difference of their Squares $xx - ee$,
14 The Sum of their Sum and Difference $2x$,
15 The Difference of their Sum and Difference $2e$
16 The Product of their Sum and Difference $xx - ee$,
17 The Square of their Sum $xx + 2xe + ee$,
18 The Square of their Difference $xx - 2xe + ee$,
19 The Square of their Product $xxee$,
20 The Cube of the greater xxx, or x^3,
21 The Cube of the leſs eee, or e^3, &c.

Theſe being underſtood, you may proceed to the working of the following *Algebraic Problems.*

DIA-

DIALOGUE X.

SECT. I.

ALGEBRAIC PROBLEMS, *or the Solution of Questions producing* SIMPLE EQUATIONS.

Between PHILOMATHES, TYRUNCULUS, *and* NOVITIUS; *being a proper Exercise of all the foregoing Rules.*

Tyr. THIS is *that* Part of *Algebra* that I have so long wish'd to be trying at, and to which by your kind Affiftance, *Philomathes*, I am at laft happily arrived to.

Phi. I am as much fatisfied, and take as great Pleafure in your Progrefs as you poffibly can, and I doubt not of your Underftanding the Manner of working the *Problems* in a fhort Time. Only take Care to mind the Steps in the *numerical* Work, and you will foon underftand the *literal*, for I fhall endeavour to make the Steps alike if I can. And though you be perfect in the Chief of what you have done, yet give me Leave once more to remind you of thefe three Things, *viz.* That this Character (φ) in any Step fhews you that the Number or Quantity before which it is placed is tranfpofed in the next Step to the other Side of the *Equation*; this (Q.) fignifies *by the Queftion*; and laftly, to remember that to take the $\frac{1}{2}$ or $\frac{1}{3}$, *&c.* of any *Fraction* is only to multiply

ply the Denominator by 2, 3, &c. which is the ſame as to divide the Numerator by the ſame Figures. Theſe being obſerved, we will proceed to

PROBLEM I.

What Number is that which being multiplied by 12, *and having* 18 *added to the Product, the Sum will be* 294?

Numerical Solution.

Put	1	x for the Number, this $\times$ 12
	2	$12x$, add 18, it is
	3	$12x + 18$. This Q. $= 294$. Then ϕ 18
	4	$12x = 294 - 18$; that is,
	5	$12x = 276$. Then is
	6	$x = \frac{276}{12} = 23$ *Anſ.*

Literal Solution.

Let $b = 12$, $c = 18$, $d = 294$.

1	x as before $\times\ b$
2	xb add $+\ c$
3	$xb + c$. This Q. $= d$
4	$xb + c = d$. Then $\phi\ c$
5	$xb = d - c$
6	$x = \frac{d-c}{b} = 23$ *Anſ.*

PROBLEM II.

What Number is that to which if I add 24, *then from that Sum ſubtract* 8, *and multiply the Remainder by* 5, *the Product will be* 320?

Numerical

Numerical Solution.

Put	1	x for the Number, then
	2	$x + 24$, then $- 8$,
	3	$x + 24 - 8$; this $\times 5$, is
	4	$5x + 120 - 40$. Then Q.
	5	$5x + 120 - 40 = 320$. By φ 40
	6	$5x + 120 = 320 + 40$. Then φ 120
	7	$5x = 360 - 120 = 240$. Therefore
	8	$x = \frac{240}{5} = 48$ *Anf.*

Literal Solution.

Let $b = 24$, $c = 8$, $d = 5$, $f = 320$.

Put	1	x as before
	2	$x + b$
	3	$x + b - c$. This $\times d$
	4	$dx + db - dc$. Whence Q.
	5	$dx + db - dc = f$
	6	$dx + bd = f + dc$
	7	$dx = f + dc - bd$
	8	$x = \frac{f + dc - bd}{d} = 48$

☞ But it is to be noted that all fuch like Queftions as *this* may be performed both fhorter and eafier, by working only with the Difference of the Numbers, and not the Numbers themfelves. Thus, you are defired in the *Problem* to add 24, then fubtract 8. Now it is evident that $24 - 8 = 16$; therefore if you work only with 16, by adding it to the unknown Quantity, it muft be the fame as to add 24 and fubtract 8. So that the *Problem* may be read thus:

PRO-

PROBLEM II. *in other Words.*

What Number is that to which if I add 16, *and multiply that Sum by* 5, *the Product will be* 320 ?

Numerical Solution.

Put	1	x as before, then $+ 16$,
	2	$x + 16$. This $\times 5$,
	3	$5x + 80$. Whence Q.
	4	$5x + 80 = 320$. Then φ 80
	5	$5x = 320 - 80 = 240$. Then
	6	$x = \frac{240}{5} = 48$, as before.

Literal Solution.

Let $b = 16$, $f = 320$, $= d$ 5

1	x
2	$x + b$. This $\times d$
3	$xd + bd$ Q.
4	$xd + bd = f$
5	$xd = f - bd$
6	$x = \frac{f - bd}{d} = 48$, as before

Tyr. This is much ſhorter and better indeed as you obſerve, and I begin to underſtand Something of the *literal* Operation, as well as the *numerical*; but I muſt needs ſay, I like the *numerical* beſt, I think it is the plaineſt for Learners.

Phi. Moſt are apt to ſay ſo indeed; but when once the other Way is known, you will like that as well, and to be ſure it is the ſhorter of the two, but I will not ſay the eaſier. However, I will perform all the *Problems* numerically, and ſome of them literally; and pray let me adviſe you to read over every

every Queſtion, at leaſt twice or thrice, that you may underſtand the Nature of it the better, for when once you have a true and juſt *Idea*, of the Intent and Meaning of it, you may be ſaid to have half done it.

PROBLEM III.

Says Alexander *to* Epheſtion, *I am older than you by two Years.* Clitus *hearing it, ſaid, I know I am older than both of you by four Years. The Philoſopher,* Calliſthenes, *being preſent, ſaid, I remember I have heard my Father, who is now ninety-ſix Years, ſay, that he is as old as you all; I demand then the Age of* Alexander, Clitus, *and* Epheſtion.

Numerical Solution.

Put
| 1 | x *Epheſtion*, then will
| 2 | $x + 2$ be *Alexander*'s. Then $+ 4$
| 3 | $2x + 6$ *Clitus*. Theſe added
| 4 | $4x + 8$ their Sum; whence Q.
| 5 | $4x + 8 = 96$. Then φ 8
| 6 | $4x = 96 - 8 = 88$. Therefore
| 7 | $x = \frac{88}{4} = 22$ *Epheſtion*'s Age
| 8 | x viz. $22 + 2 = 24$ *Alexander*'s
| 9 | $xx + 6 = 50$ *Clitus*'s

Literal Solution.

Let $b = 2$, $d = 4$, $f = 96$
| 1 | x *Epheſtion*'s,
| 2 | $x + b$ *Alexander*'s,
| 3 | $2x + b + d$ *Clitus*'s,
| 4 | $4x + 2b + d$ Sum. Then Q.
| 5 | $4x + 2b + d = f$. Then φ $2b + d$
| 6 | $4x = f - 2b - d$

| 7 |

7	$x = \frac{f - 2b - d}{4} = 22$ *Epheſtion*'s
8	$x + b = 24$ *Alexander*'s
9	$2x + b + d = 50$ *Clitus*'s.

Tyr. Mighty pretty; but why do you begin with *Epheſtion* rather than *Alexander?*

Phi. It would have been the ſame had I began with *Alexander*'s Age; only then the 2*d* Step would have been $x-2$ for *Epheſtion*, and the 3*d* Step $2x-2+4$; and ſo it would have occaſioned more Work, but now they are all *affirmative*.

Tyr. I am ſatisfied, and begin to ſee a little more into it.

Phi. There is no Fear of your Underſtanding it, if you mind.

PROBLEM IV.

Three Perſons, A, B, *and* C, *trade and gain* 3000 *£. the Share of* A *is to be but Half the Share of* B, *and the Share of* B *one Third the Share of* C; *I demand each Man's Share?*

Now to avoid *Fractions*, I begins with *A* firſt; for if I put x for *B*, then *A* muſt be $\frac{x}{2}$ and *C* $3x$. Therefore

Numerical Solution.

Put	1	x for *A*, then is
	2	$2x$ *B*'s Share, and
	3	$6x$ *C*'s Share. Theſe added make
	4	$9x$ their Sum. Whence Q.
	5	$9x =$ £. 3000. Therefore

6	$x = £.\ \frac{3000}{9} = £.\ 333\ 6\,s.\ 8\,d.\ A,$
7	And by the 2*d* Step 666 13 *s.* 4 *d.* *B*,
8	And by the third 2000 *C*,
9	Their Sum $=$ 3000.

Literal Solution.

	As x is put for *A*, and the other is double and treble, the firſt four Steps will be the ſame as in the *numerical*. Now let
	$b = £.\ 3000$
5	$9x = b$
6	$x = \frac{b}{9} = £.\ 333\frac{1}{3}$
7	$2x = 666\frac{2}{3}$
8	$6x = 2000$
9	Sum $=$ 3000 £. as before.

Do yo underſtand theſe Operations?

Tyr. The *literal* Part I am not at preſent ſo much acquainted with, but the *numerical* appears quite plain and eaſy to me: I heartily wiſh *Novitius* was here, he would be ſo pleas'd to ſee ſome of thoſe Queſtions which puzzle him, demonſtrated in ſo eaſy a Manner.

Phi. —— There is a young Gentleman now coming up the Walk.

Tyr. Perhaps it is he———It is ſo—I will go to the Door for I know he is quite baſhful——

Phi. Stay——give me Leave, *Tyrunculus*; it will look better in me, and he will take it kinder at my Hands.———

Nov. Your humble Servant, *Philomathes*: Pray is *Tyrunculus* here?

Phi. He is, pray come in, *Novitius*.

Nov.

Nov. Sir——Friend *Tyrunculus*, how fare you?

Tyr. I am a little vexed with you for ſtaying.

Nov. I aſk *Philomathes*'s Pardon in particular—I was unexpectedly prevented by an Acquaintance.

Phi. Well, *Novitius*, we will not uſe ſuperfluous Ceremonies at this Time: Pray ſit down, I am glad to ſee you.

Nov. Sir I thank you.

Phi. Your Friend *Tyrunculus* was ſaying you had ſome Queſtions to aſk me that had puzzled you pretty much, pray, what are they?

Nov. Only ſome few of Mr. *Cocker*'s and *De-Billy*'s, for ſeveral of them are ſo contracted in the Work that I cannot underſtand them.

Phi. To do Juſtice to the firſt Author, I know not a prettier Piece for Learners on the firſt four Rules of *Algebra*; but I confeſs he is a little dark in ſome of the Operations. Come, I have him by me, and pray do you look out thoſe Queſtions that puzzle you moſt, and we will work them more plainly to your Underſtanding.

Nov. Pleaſe then to begin with his fifth Queſtion.

Phi. There are *Fractions* concerned in that; therefore I think we had better begin with the more eaſy ones firſt, and take the harder as they come in Courſe.

Nov. It is true; do ſo if you pleaſe.

Phi. Obſerve then.

* PROBLEM V. — *Cocker*'s 8*th* Queſtion.

A Labourer received 2 £. 8 *s. for threſhing* 60 *Quarters of Corn,* viz. *Wheat and Barley; for the Wheat he received* 12 *Pence a Quarter, and for the Barley* 6 *Pence: How many Quarters did he threſh of each.*

Numerical Solution.

1 | For the Quarters of Wheat put x,
2 | Then as both together are but 60, the Barley muſt be $60 - x$.
3 | Now x Quarters of Wheat, at 12*d.* a Quarter, is $12x$.
4 | And $60 - x$ Quarters of Barley, at 6*d.* is $360 - 6x$.
5 | Theſe two are equal Q. to 48*s.* or 576 Pence; whence $12x + 360 - 6x = 576$.
6 | Then ϕ 360, $12x - 6x = 576 - 360$;
7 | That is, $6x = 576 - 360 = 216$.
8 | Therefore $x = \frac{216}{6} = 36$ Quarters of Wheat.
9 | And by 2*d* Step $60 - x$, or $60 - 36 = 24$ the Barley.

Literal Solution.

Let $b = 576$ Pence, $c = 60$, $d = 12$, $f = 6$ Pence.
1 | x Wheat.
2 | $c - x$ Barley. Then $x \times d$ is
3 | dx for the Wheat, and $f \times \overline{c - x}$ is

| 4 |

*Those *Problems* that have an Aſteriſk before them, are inſerted by the Deſire of ſeveral young *Algebraiſts*, who wanted a plainer and eaſier Demonſtration than in the Original: And to ſuch as are unacquainted with either of the Authors, they will be equally ſerviceable to, as if they were new ones.

4 | $fc - fx$ for the Barley. Whence Q.
5 | $dx + fc - fx = b$, or 576 Pence. Then
6 | $dx - fx = b - fc$. Therefore
7 | $x = \frac{b - fc}{d - f}$; that is, $x = \frac{576 - 360}{12 - 6} = 36$ as above Wheat,
8 | And $c - x$, or $60 - 36 = 24$, the Barley.

PROOF.

36 Quarters, at 1 *s.* = 36 *s.*
24 Quarters, at 6 *d.* = 12 *s.*

48 *s.*

Nov. I underſtand it quite well.

Tyr. So do I both the Ways.

Phi. I ſhall not write againſt the *literal* any more, but leave you to compare it with the *numerical* Work.

* PROBLEM VI. — *Cocker*'s 10*th* Queſtion.

A Gentleman hired a Servant for 40 *Days upon this Condition, that every Day he wrought he was to receive* 20 *Pence, and for every Day he was idle, and did no Work, he was to pay* 8 *Pence. Now at the End of the Time he received* 15s. 4d. *How many Days did he work, and how many was he idle?*

Numerical Solution.

1 | For the Days he wrought put x
2 | Then will what he play'd be $40 - x$.
3 | Now x Days wrought, at 20 *d.* a Day, is $20x$,
4 | And $40 - x$ play'd, at 8 *d.* a Day, is $320 - x$.
5 | Subtract the 4*th* Step from the 3*d* Step, it is $20x - 320 + 8x$.

6 | This being his Due, is (Q.) $= 15$ *s.* 4 *d.*
Whence $20x - 320 + 8x = 184$ *d.*
7 | That is, $28x - 320 = 184$,
8 | That is, $28x = 184 + 320 = 504$.
9 | Therefore $x = \frac{504}{28} = 18$ Days *Anſ.*
10 | And by 2*d* Step, $40 - x = 22$ idle.

Literal Solution.

Let $b = 40$, $c = 20$, $d = 8$, and $f = 184$ Pence, *viz.* 15*s.* 4*d.*

1 | x Days wrought,
2 | $b - x$
3 | cx
4 | $db - dx$. Whence
5 | $cx - db + dx = f$
6 | $cx + dx = f + db$
7 | $x = \frac{f + db}{c + d}$ 18, as before;
8 | $b - x = 22$ idle.

PROOF.

	£.	s.	d.
18 Days, at 20 *d.* a Day,	1	10	0
22 idle, at 8 *d.* a Day,	0	14	8
Due at laſt	0	15	4

* PROBLEM VII. — *Cocker*'s 6*th* Queſtion.

Two Perſons, A *and* B, *thus diſcourſed of their Money: Says* A *to* B, *give me* 3 *of your Crowns, and I ſhall have as many as you; and ſays* B *to* A, *give me* 3 *of yours, and I ſhall have* 5 *Times as many as you. I demand the Number each had?*

Numerical

Numerical Solution.

1 For the N°. *A* had at firſt put x,
2 Then by *B*'s giving him 3, he will have $x+3$;
3 And as this makes *B*'s equal, *B* will alſo have $x+3$,
4 And therefore conſequent *B* had at firſt $x+6$,
5 And if *A* had given him 3, he would then have $x+9$
6 And by the ſame Reaſon *A* would have but $x-3$
7 Now *B* (Q.) ſhould have 5 Times this Number, *viz.* $5x-15$
8 Whence this *Equation*, $5x-15=x+9$.
9 Then ϕ x and -15, it will be $5x-x=9+15$,
10 That is $4x=24$.
11 Therefore $x=\frac{24}{4}=6$ *A*'s.
12 And by the 4*th* Step $x+6=12$ *B*'s.

Literal Solution.

Let $e=3$ Crowns; then by comparing the Steps,
1 x for *A*,
2 $x+e$,
3 $x+e$,
4 $x+2e$,
5 $x+3e$,
6 $x-e$,
7 $5x-5e$
8 $5x-5e=x+3e$,
9 $5x-x=3e+5e$,
10 $4x=8e$

11	$x = \frac{8e}{4} = 6$, *A*'s.
12	$x + 2e, = 12$, *B*'s.

PROOF.

$$A\ 6 + 3 = 9,$$
$$B\ 12 - 3 = 9, \text{ but}$$
$$A\ 6 - 3 = 3, \text{ and}$$
$$B\ 12 + 3 = 15 = 3 \times 5.$$

Nov. I am perfectly ſatisfied, it is done ſo plain, and the Steps are built upon Reaſon itſelf.

Phi. It is upon Reaſon itſelf that *Algebra* depends; and a Queſtion laid down in a good and clear Light, is the Learners chief Guide. But now for ſome that require *Fractions*, for I ſuppoſe you underſtand them!

Nov. Yes, I think I do pretty well.

Phi. Here follows then.

* PROBLEM VIII. — *Cocker*'s 5th.

There is a Fiſh whoſe Head is 9 Inches long, and his Tail is as long as his Head, and half as long as his Body, and his Body is as long as his Tail and his Head; I demand the whole Length of the Fiſh.

1	For the Length of his Body put x
2	Then will his Tail be $\frac{x}{2} + 9$,
3	And his Body ſhould be as long as his Tail and Head, *viz.* $\frac{x}{2} + 9 + 9$.
4	Whence (Q) this Equation $x = \frac{x}{2} + 18$,

| 5 |

5 This $\times$ Denominator 2, is $2x = x + 36$.
6 Then ϕ x, $2x - x = 36$,
7 That is, $x = 36$ his Body,
8 And by 2*d*. Step, $\frac{x}{2} + 9 = 27$ his Tail;
9 To which add 9 his Head,
10 Their Sum is 72 his Length.

Literal Solution.

Let $b = 9$,

1 x,
2 $\frac{x}{2} + 2b$,
3 $\frac{x}{2} + 2b$,
4 $x = \frac{x}{2} + 2b$,
5 $2x = x + 4b$,
6 $2x - x = 4b$,
7 $x = 4b = 36$, Body.
8 $\frac{x}{2} + b = 27$, Tail.

PROBLEM IX. His 12*th* Queſtion.

One aſked a Shepherd the Price of his 100 *Sheep. No, replied he, if I had as many more, and half as many more, and ſeven Sheep and an half, I ſhould then have* 100. *I demand the Number he had.*

Numerical Solution.

Put
1 x for the Number, then
2 x as many more, and
3 $\frac{x}{2}$ is half as many more, and

4	$7\frac{1}{2}$. These four Steps added
5	$2x + \frac{x}{2} + 7\frac{1}{2}$. Whence (Q.)
6	$2x + \frac{x}{2} + 7\frac{1}{2} = 100$. This reduced
7	$4x + x + 15 = 200$. Then ϕ 15,
8	$5x = 200 - 15 = 185$. Therefore
9	$x = \frac{185}{5} = 37$ *Answer.*

Literal Solution.

Let $b = 7\frac{1}{2}$, $c = 100$.

1	x,
2	x,
3	$\frac{x}{2}$,
4	b,
5	$2x + \frac{x}{2} + b$ Sum (Q.)
6	$2x + \frac{x}{2} + b = c$,
7	$4x + x + 2b = 2c$,
8	$5x = 2c - 2b$,
9	$x = \frac{2c - 2b}{5} = 37$. *Ans.*

* PROBLEM X. His 9*th* Question.

A Gentleman bought a Cloak of a Salesman, which cost him 3£. 10*s. and after he had bought it he desired the Salesman to tell him ingenuously what he gain'd by it, who answered I gain just* $\frac{1}{4}$ *Part of what it cost me. It is demanded what the Cloak cost the Salesman.*

Numerical Solution.

1	For what the Cloak cost the Salesman put x,
2	Then his Gain or Profit will be $\frac{x}{4}$

| 3 |

3 | These added are equal to what he sold it for, $x + \frac{x}{4}$,

4 | Therefore (Q.) $x + \frac{x}{4} = 70s.$

5 | This reduced, $4x + x = 280$;

6 | That is, $5x = 280$.

7 | Therefore $x = \frac{280}{5} = 56s.$ *Ans.*

Literal Solution.

Let $e = 70$.

1 | x,

2 | $\frac{x}{4}$,

3 | $x + \frac{x}{4}$

4 | $x + \frac{x}{4} = e$,

5 | $4x + x = 4e$,

6 | $5x = 4e$,

7 | $x + \frac{4e}{5} = 56$ as before.

PROOF.

Cost him $56\,s.$
Gained $\frac{1}{4} = 14\,s.$

Sum $= 70s.$ sold it for.

* PROBLEM XI. — His 11*th* Question.

A Person in the Afternoon being ask'd what a Clock it was, answered, that $\frac{3}{5}$ of the Time past from Noon, was equal to $\frac{5}{8}$ of the Time to Midnight. Now allowing

allowing 12 Hours to the Day, and beginning to reckon from Noon, I demand what Hour it was when the Question was asked.

Numerical Solution.

1 | For the Hours sought from Noon put x,
2 | Then will the Time to Midnight be $12 - x$,
3 | Now $\frac{3}{5}$ of x is $\frac{3x}{5}$,
4 | And $\frac{5}{8}$ of $12 - x$ is $\frac{60 - 5x}{8}$.
5 | These (Q.) are equal. Whence $\frac{60 - 5x}{8}$.
6 | This reduced, *viz.* first $\times$ Denominator 5, is $3x = \frac{300 - 25x}{8}$.
7 | Then this $\times$ the Denominator 8 will be $24x = 300 - 25x$,
8 | Then $\varphi - 25x$ it is $24x + 25x = 300$,
9 | That is, $49x = 300$.
10 | Therefore $x = \frac{300}{49} = 6\frac{6}{49}$.
11 | And by 2*d* Step $12 - x = 5\frac{43}{49}$.

So that it was $\frac{6}{49}$ past 6, that is $7'\ 20''\ \frac{40}{49}$ past 6.

PROOF.

	H.	′	″	
$6\frac{6}{49}$ =	6	7	20	$\frac{40}{49}$
$5\frac{43}{49}$ =	5	52	39	$\frac{9}{49}$.

Sum = 12 Hours.

Literal

Literal Solution.

Let $b = \frac{3}{5}$, $c = \frac{5}{8}$, and $d = 12$

1	x as above,
2	$d - x$,
3	bx,
4	$cd - cx$,
5	$bx = cd - cx$,
6	$bx + cx = cd$,
7	$x = \frac{cd}{b+c}$, viz. $x = \frac{5}{8} \times 12 \div \overline{\frac{3}{5} + \frac{5}{8}} = 6 \frac{48}{392}$ or $6 \frac{6}{49}$ as before.

Nov. This is plain upon my Word, and I perceive $6 \frac{48}{392}$ is the fame Anfwer as Mr. *Cocker*'s; but ftill I never could know, nor do I yet rightly apprehend from whence this $\frac{48}{392}$ proceeds; and therefor I never could make the Anfwer chime in with his.

Phi. I muft own that a Man had need to underftand *Fractions* quite perfectly to find fuch Things out to his own Satisfaction; for it is not every *Learner* can do it. Obferve then, the Anfwer is $x = \frac{5}{8} \times 12 \div \frac{3}{5} + \frac{5}{8}$. Now $\frac{5}{8} \times \frac{12}{1} = \frac{60}{8}$. And $\frac{3}{5} + \frac{5}{8}$ being reduced to a common Denominator and added, their Sum will be $\frac{24}{40}$ and $\frac{25}{40}$ (that is $\frac{49}{40}$) by which divide $\frac{60}{8}$ it is $\frac{2400}{392} = 6 \frac{48}{392}$ as before; and therefore the Hour to Midnight is $5 \frac{344}{392} = 5 \frac{43}{49}$.

Tyr. There, *Novitius*, is that plainly demonftrated or not?

Nov. Quite plain indeed! And now *Philomathes* I will afk you to work the 6*th* Queftion and no more, for the 5*th* Step, I never could apprehend it, it has puzzled me many a Time.

Phi. I muſt confeſs I was a long Time in finding it out myſelf, and tho' the ſame Queſtion be in Mr. *Ward*, it did not anſwer my Expectation; for I found it out by Mr. *Cocker*'s Method at laſt.

Nov. But then he abbreviates his Numbers and *Fractions*, and gives no Reaſon, and this puts the Learner to a Stop. *

Phi. It is true he does ſo. Well, *Novitius*, all I can ſay, is, I will work the ſame Queſtion *numerically*, and leave you to judge which of the three Ways are the eaſieſt to be underſtood, ſuppoſing you had them now all before you.

* PROBLEM XII. *Cocker*'s 6*th*, and *Ward*'s 31*ſt* Queſtion.

A Father lying at the Point of Death, left his three Son, A, B, *and* C, *his Eſtate as follows. To* A *he gave* $\frac{1}{2}$ *wanting* 44 £. *to* B *he gave* $\frac{1}{3}$ *and* 14 £. *over; and to* C *he gave the Remainder which was* 82 £. *leſs than the Share of* B. *I demand the Father's Eſtate in ready Money?*

Numerical Solution.

1	For the Eſtate put x,
2	Then will *A*'s Legacy be $\frac{x}{2} - 44$,
3	And *B*'s will be $\frac{x}{3} + 14$,
4	And *C*'s being 82 leſs, is $\frac{x}{3} + 14 - 82$.
5	Theſe Fractions being firſt reduced to a common Denominator, and added, the Sum of the Whole is $\frac{2\ x}{18}$, or $\frac{7x}{6} + 28 - 126$. This (Q.) equal to the Eſtate or x. [6]

* See the Preface.

6	Whence this Equation $\frac{7x}{6} + 28 - 126 = x$,
	Now 28—126 being the ſame as—98, it will
7	be $\frac{7x}{6} - 98 = x$.
8	This reduced $7x - 588 = 6x$.
9	Then φ $6x$, and 588, it will be $7x - 6x = 588$,
10	That is, $x = 588$ Eſtate.

PROOF.

A's Share $\frac{1}{2} - 44 = 250 = \frac{x}{2} - 44$ by 2*d* Step.

B's $\frac{1}{3} + 14 = 210 = \frac{x}{3} + 14$ by 3*d*. Step.

C's 82£. leſs than *B*, $= 128 = \frac{x}{3} + 14 - 82$ by 4*th* Step.

$$\overline{588}$$

Nov. I heartily thank you, kind *Philomathes*; then I perceive that Mr. *Cocker*'s 5*th* Step before *Abbreviation*, was $\frac{21a}{18} + 2c - b - d$.

Phi. You are right, and one would think a Learner might eaſily perceive it if he would be diligent.

Nov. You know, Sir, a ſmall Matter turns the Learner quite out of his direct Path. I have now with me *James de Billy*'s *Algebra*, but it is wrought in ſo odd a Manner, that I cannot make out any Thing by it. I could wiſh you would work ſome of his Queſtions *numerically*.

Phi. Which would you have me begin with?

Nov. His 2*d* Queſtion if you pleaſe.

Phi. I will work ſeveral of them.

* PRO-

* PROBLEM XIII. — *J. de Billy*'s 2*d* Queſtion.

A Hare is 100 *Yards diſtant from a Dog, and both ſtarting together, the Dog ran* 2 ½ *Times faſter than the Hare: It is demanded how far the Hare will have run before the Dog overtakes her?*

Numerical Solution.

1 For the Yards the Hare ran put x,

2 Then will the Dog when he overtakes her have run $100 + x$.

3 Now becauſe he runs 2 ½ Times faſter than the Hare, take any two Numbers bearing the like Proportion as 5 and 2, then.

4 By the Rule of 3, as $5 : 2 :: 100 + x : x$,

5 Multiplying *Means* and *Extremes*, $5x = 200 + 2x$.

6 Then φ $2x$, — — — $5x - 2x = 200$,

7 That is, — — — $3x = 200$.

8 Therefore $x = \frac{200}{3} = 66\frac{2}{3}$

9 And by 2*d* Step, $100 + x = 166\frac{2}{3}$.

So that the Hare run $66\frac{2}{3}$ Yards, and the Dog $166\frac{2}{3}$.

* PROBLEM XIV. — His 11*th* Queſtion.

A certain Man agreed with his Servant for 12 *Months Service to give him* 10 *Crowns and a new Coat; but diſagreeing, he at the End of* 7 *Months gives him the Coat and two Crowns: I demand the Value of the Coat?*

Numerical

Numerical Solution.

1 For the Value of the Coat put x,

2 Then will this and 10 Crowns be his Year's Wages, $x + 10$.

3 Now to find one Month's Wages say, As $12 : x + 10 :: 1 : \frac{x + 10}{12}$,

4 And becauſe he had 2 Crowns and the Coat for 7 Months, ſay,

As $7 : x + 2 :: 1 : \frac{x + 2}{7}$.

5 Theſe being both 1 Month's Wages are equal. Whence $\frac{x + 2}{7} = \frac{x + 10}{12}$.

6 This $\times$ the Denominator 7, is $x + 2 = \frac{7x + 70}{12}$.

7 This $\times$ 12, the other Denominator, $12x + 24 = 7x + 70$, then φ $7x$ and 24,

8 $12x - 7x = 70 - 24$, that is,

9 $5x = 46$,

10 Therefore $x = \frac{46}{5} = 9\frac{1}{5}$ Crowns.

So that the Value of the Coat was $9\frac{1}{5}$ Crowns, or 46 Shillings, to which add two Crowns, is 56 Shillings that he had for his 7 Month's Service; and for the Year's Service, had he ſtaid, $x + 10$ Crowns by 2*d* Step, *viz.* 96 Shillings.

* PROBLEM XV. — His 16*th* Queſtion.

A Man having a certain Number of Crowns about him, deſired a Stander-by to gueſs at them, who ſaid, you have 600 *perhaps. No, ſays he; but if to what I have were added* $\frac{1}{2}$, $\frac{1}{3}$, *and* $\frac{1}{4}$, *and from what I have*

have were ſubtracted $\frac{1}{12}$, *I ſhould then have* 600: *I demand the Number he had about him?*

Note, As in *Problem* 2, ſo here alſo make the Subtraction firſt, and work with the Difference only. Thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, reduced to a common Denominator, will be $\frac{26}{24}$, or $1\frac{2}{24} = 1\frac{1}{12}$; from which take $\frac{1}{12}$, there remains 1 whole Integer. So that ſuppoſing I were to put x for the Number, then $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ added, and $\frac{1}{12}$ taken from it, x will be the Difference; therefore $2x = 600$, and $x = 300$ the Number. But ſee the Whole as follows.

1	For the Number ſought put x,
2	Then $\frac{x}{2}, \frac{x}{3}, \frac{x}{4} = \frac{25x}{24}$, that is, $= x + \frac{x}{12}$ as before.
3	Add theſe, it is $2x + \frac{x}{12}$,
4	From which take $\frac{1}{12}$, *viz.* $\frac{x}{12}$,
5	There remains only $2x$. Whence (Q.) $2x = 600$.
6	And therefore $x = 300$ *Anſ.*

Nov. I heartily thank you, this appears plain enough to me. I will trouble you to work one more if you pleaſe, and that is his 7*th* Queſtion, for this is quite dark to me.

* PPOBLEM XVI. – His 7*th* Queſtion.

A certain Perſon bought a Number of Ells of Velvet, which he ſold again; he bought 5 *Ells for* 7 *Crowns, and ſold ſeven Ells for* 11 *Crowns, and gained* 100 *Crowns in ſo doing: I demand how many Ells there were in all?*

Numerical

Numerical Solution.

1 | For the Number put x,
2 | Then, if 5 Ells be 7 Crowns, what will x be? $\frac{7x}{5}$,
3 | Then, if 7 Ells be ſold for 11 Crowns, what will x fetch? — — $\frac{11x}{7}$,
4 | The Difference of theſe ſhew his Gain $\frac{6x}{35}$. *
5 | This (Q.) $=$ 100 Crowns. Whence $\frac{6x}{35} = 100$.
6 | This reduced $6x = 3500$.
7 | Therefore $x = \frac{3500}{6} = 583\frac{1}{3}$.
So that the Number was $583\frac{1}{3}$ Ells boug[ht] and ſold.

PROOF.

If 5 Ells be 7 Crowns, what is $583\frac{1}{3}$? *Anſ.* $816\frac{1}{3}$ Cr.
If 7 Ells be 11 Crowns, what is $583\frac{1}{3}$? *Anſ.* $916\frac{1}{3}$ Cr.

He gained 100 Cr.

PROBLEM XVII.

There are two Numbers whoſe Sum is 240, *and the Greater has the ſame Proportion to the Leſs as* 7 *to* 3: *I demand the Numbers?*

Numerical Solution.

1 | For the Greater put x,
2 | Then will the Leſs be $240 - x$.

| 3 |

* Note, *The Fractions* $\frac{7x}{5}$ *and* $\frac{11x}{7}$ *are firſt reduc'd to a common Denominator, and then the Difference you will find is* $\frac{6x}{35}$.

3 | These (Q.) have this Proportion, as $7 : 3 :: x :: 240 - x$.
4 | Then multiplying *Means* and *Extremes*, $3x = 1680 - 7x$.
5 | Then $\phi - 7x$, it is $3x + 7x = 1680$.
6 | That is, $10x = 1680$.
7 | Therefore $x = \frac{1680}{10} = 168$ G. Number.
8 | And by 2*d* Step $240 - x = 72$ L. Number.

* PROBLEM XVIII.

A certain Toper went to an Alehouse, and borrow'd as much Money as he had about him, out of which he spent a Shilling; then he went to a 2d *Alehouse, and borrowed as much as he had then about him, and spent a Shilling; and in like Manner he went to a* 3d *and* 4th *Alehouse, borrowing as much as he had left at the former, and spent a Shilling; but after he had spent a Shilling at the 4th Alehouse he had Nothing left: It is demanded what he had first about him?*

Numerical Solution.

1 | For what he first had put x,
2 | Then by borrowing as much, he had $2x$,
3 | And when he had spent 12 Pence, had $2x - 12$.
4 | Then by borrowing the same, had $4x - 24$,
5 | And by spending 12 Pence, had $4x - 36$;
6 | Then, by borrowing the same, had $8x - 72$,
7 | And by spending 12 Pence, had $8x - 84$.
8 | Then by borrowing the same, had $16x$ 168,
9 | Then by spending 12 Pence, had left $16x - 180$,
10 | That is, had Nothing left. Whence (Q.) $16x - 180 = 0$,
11 | That is, $16x = 180$.

| 12 |

12	Therefore $x = \frac{180}{16} = 11\frac{4}{16}$
	So that he had at firſt 11 *d.* $\frac{1}{4}$.

* PROBLEM XIX.

One being aſked how old he was, anſwered thus:

If to the Number of my Age you add
The one Half of three Fourths, and 14 *more,*
The Number 58 *will then be had:*
What is my Age in Years above a Score?

Note, As the Queſtion ſays, if you add 14 to his Age, it will make 58; ſo conſequently without adding the 14 it will be 44; therefore working with 44, without adding 14, will be better than to work with 58 and 14 together, as you may obſerve in *Problem* 3*d*.

Numerical Solution.

1	For his Age put x,
2	Then by adding $\frac{1}{2}$ of $\frac{3}{4}$, that is, $\frac{3}{8}$, it is $x + \frac{3x}{8}$.
3	This (Q.) $= 44$. Whence $x + \frac{3x}{8} = 44$.
4	This reduced $8x + 3x = 352$.
5	That is, $11x = 352$.
6	Therefore $x = \frac{352}{11} = 32$.
	So that his Age was 32, 12 above a Score.

PROBLEM XX.

A Citizen riding his Rounds to receive Money due to him, came to a Place in which he had three Debtors, A, B, *and* C, *but when he came to ex-*

 amine,

amine, he found he had lost his Pocket-Book, in which was each Man's separate Bill. Then sending for them all to an Inn, he tells them the Accident; but they pretended they did not know what was due to him, having lost the Bills that came with the Parcels. The Gentleman thinking he had got some slippery Chaps to deal with, endeavoured all he could to save himself the Trouble of another Journey; and this he did from the following Data: *He remembered very well that* A's *and* B's *Debt together made* 13 £. 10 *s. and* A's *and* C's *Debt together* 31 £. 10 *s. and* B's *and* C's *together made* 37 £. 10 *s. It is demanded what each Man's particular Debt was?*

1 | For *A*'s Debt put x,
2 | Then as *A*'s and *B*'s together is 270 *s.* *B*'s is $270 - x$,
3 | Then as *A*'s and *C*'s is 630 *s.* *C*'s alone is $630 - x$.
4 | Now (Q.) *B*'s and *C*'s should be 750 *s.* but is $900 - 2x$.
5 | Whence this Equation $750 = 900 - 2x$.
6 | By φ $2x$, — — — — $750 + 2x = 900$.
7 | And φ 750, — — — $2x = 900 - 750$,
8 | That is, $2x = 150$.
9 | Therefore $x = \frac{150}{2} = 75$ *s.* *A*'s
10 | And by 2*d* Step, $27\,s. - x = 195$, *B*'s.
11 | And by 3*d* Step, $630 - x = 555$, *C*'s.

	£.	s.
So that *A*'s Debt was	3	15
B's — — —	9	15
C's — — —	27	15

* PRO-

PROBLEM XXI.

A Highwayman robb'd a Gentleman of a certain Sum of Money, but being ſeen by 3 Men, A, B, *and* C, *they purſued and took him; but he promiſed to make them a handſome Preſent if they would let him go; to which they agreed. To* A *he gave* $\frac{1}{2}$, *and* A *return'd him back* 6 £. *To* B *he gave* $\frac{1}{3}$ *of what was left, who return'd him* 4 £. *And to* C *he gave* $\frac{1}{4}$ *of what he had then left, who return'd him back* 2 £. *And after he had rode off, and came to tell his Money, he found he had given* $\frac{2}{3}$ *of it away: I demand the Sum he took from the Gentleman?*

Numerical Solution.

1 | For what he took in Pounds put x,

2 | Then by giving $\frac{1}{2}$ he had the ſame left, $\frac{x}{2}$.

3 | And by *A*'s returning him 6, he had $\frac{x}{2}+6$

4 | This he gave *B* $\frac{1}{3}$ of, *viz.* $\frac{x}{6}+2$,

5 | And conſequently had himſelf $\frac{2}{3}$, *viz.* $\frac{2x}{6}+4$, or $\frac{x}{3}+4$,

6 | And by *B*'s returning 4, had $\frac{x}{3}+8$.

7 | This he gave *C* $\frac{1}{4}$ of, *viz.* $\frac{x}{12}+2$,

8 | And conſequently had in Hand $\frac{3}{4}$, *viz.* $\frac{3x}{12}+6$,

9 | And by *C*'s returning 2, had $\frac{3x}{12}+8$, or $\frac{x}{4}+8$.

10 This (Q.) $= \frac{1}{3}$ of what he ſtole. Whence $\frac{x}{3} = \frac{x}{4} + 8$.

11 Then multiplying by the Denominator 3, $x = \frac{3x}{4} + 24$.

12 This $\times$ the Denominator 4, $4x = 3x + 96$.

13 Then φ $3x$, it will be $4x - 3x = 96$,

14 That is, $x =$ £. 96 *Anſ.*

So that he ſtole £. 96; of which *A* had 42, *B* 14, and *C* £. 8, which together make £. 64 $= \frac{2}{3}$ of 96.

PROBLEM XXII.

One being aſked how many Children he had living, anſwered 3 Times as many as he had buried; and being aſked how many that was, ſaid, that if the Number he had loſt was multiplied by the $\frac{1}{9}$ Part of what remained, it would be equal to the Number he had at firſt: I demand how many he had loſt, and how many he had left?

Numerical Solution.

1 For what he had loſt put x,

2 Then will he have left $3x$,

3 And conſequently had at firſt $4x$.

4 Now $\frac{1}{9}$ of what was left is $\frac{3x}{9}$, or $\frac{x}{3}$.

5 This multiplied by the loſt, *viz.* x, is $\frac{xx}{3}$.

6 This (Q.) equals his Number at firſt. Whence $\frac{xx}{3} = 4x$.

7 This reduced, $xx = 12x$.

8 Then dividing by x, —— — $x = 12$, loſt.
9 And by 2*d* Step, $3x = 36$, left.
10 Their Sum by 3*d* Step is $4x = 48$, at firſt.

PROBLEM XXIII.

One being aſk'd how many Teeth he had, to avoid a direct Anſwer ſaid, that he had loſt the $\frac{1}{8}$ Part of what he then had, and being aſk'd how many that was, ſaid, that if what he had loſt were multiplied into the $\frac{1}{4}$ of what he had left, and the Square of what he had left was added to that Product, it would be equal to the Number he had at firſt. Or otherwiſe, if what he had loſt were multiplied by $\frac{1}{4}$ of what he had left, it would make juſt $\frac{2}{3}$ of the Number he had at firſt: I demand how many he had loſt, and how many he had left?

Numerical Solution.

1 For the Number loſt put x,
2 Then as thoſe left are 8 Times as many, $8x$,
3 Theſe two added make the Number at firſt $9x$.
4 Now $\frac{1}{4}$ of the Number left is $2x$,
5 This multiplied by thoſe loſt, make $2xx$,
6 To which add the Square of thoſe loſt, it is $2xx + xx$.
7 Theſe (Q.) are equal to the Number at firſt.
8 Then dividing each Side by x, $2x + x = 9$.
9 That is $3x = 9$.
10 Therefore $x = 3$, loſt.
11 And by 2*d* Step, $8x = 24$, left.
12 And by 3*d* Step, $9x = 27$, at firſt.
Which you may prove according to the Tenor of the Queſtion.

PROBLEM XXIV.

An Usurer put out a certain Sum of Money at 5 £. per Cent. per Annum, *which in* 16 *Years wanted exact* 11 *Guineas of the Principal itself: I demand what the Principal was?*

Numerical Solution.

1 | For the Principal put x,

2 | Then say, If 100 be 5, what will x be? *Ans.* $\frac{5x}{100}$.

3 | This being 1 Year's Interest, 16 Years, is $\frac{80x}{100}$, or $\frac{8x}{10}$, or $\frac{4x}{5}$.

4 | This should (Q.) be equal to the Principal less 11 Guineas, or 231 Shillings. Whence, $x - 231 = \frac{4x}{5}$.

5 | This being reduced, $5x - 1155 = 4x$,

6 | Then ϕ $4x$, — — — $5x - 4x = 1155$,

7 | That is, $x = 1155$ s. $=$ £. 57, 15 s.

So that the Principal was £. 57, 15 s. which in 16 Years, at 5 *per Cent.* amounts to £. 46. 4 s. which wants 11 Guineas of the Principal.

PROBLEM XXV.

An Usurer put out 135 £. *in* 2 *Parcels, one at the Rate of* 5 per Cent. per Annum, *and the other at* 6 per Cent. *which amounted in* 15 *Years Time to the Principal itself wanting* 30 £. *I demand the Parcel he put out at* 5, *and the Parcel he put out at* 6 per Cent.

Numerical

Numerical Solution.

1 For the Parcel at 5 *per Cent.* put x,

2 Then will that at 6 *per Cent.* be $135 - x$,

3 Then, by the Rule of 3, one Year's Intereſt of x is $\frac{5x}{100}$,

4 And by the ſame Rule, one Year's of $135 - x$ is $\frac{810 - 6x}{100}$,

5 The Sum of the Intereſt of both for 1 Year is $\frac{5x}{100} + \frac{810 - 6x}{100}$.

6 Then becauſe there is $5x$ and $-6x$, it will be $\frac{810 - x}{100}$.

7 This being 1 Year's Intereſt of both, 15 Years will be $\frac{12150 - 15x}{100}$.

8 This (Q.) is equal to the Principal except £.30. Whence $105 = \frac{12150 - 15x}{100}$.

9 This × the Denominator 100, is $10500 = 12150 - 15x$.

10 Then $\phi - 15x$ and 10500, $15x = 12150 - 10500$,

11 That is, $15x = 1650$.

12 Therefore, $x = \frac{1650}{15} =$ £. 110.

13 And by 2*d* Step, $135 - x =$ £. 25.

So that he put out £. 110 at 5, and £. 25 at 6 *per Cent.* the Intereſt of which in 15 Years amounts to £. $105 = 135 - 30$.

PRO-

PROBLEM XXVI.

It is required to pay 100 £. *in* 100 *Pieces,* viz. *some to be* 15 *Shillings, and others* 22 *Shillings and* 6 *Pence each: I demand how many there must be of each Sort?*

Numerical Solution.

1	Put for the 15 *s.* Pieces x,
2	Then will those at 22 *s.* 6 *d.* be $100 - x$,
3	Now x Pieces, at 15 *s.* or 180 *d.* each is $180x$,
4	And $100 - x$ Pieces at 22 *s.* 6 *d.* or 270 *d.* each, is $27000 - 270x$.
5	The Sum of these two is $27000 - 90x$.
6	This (Q.) = £. 100, or 24000 *d.* Whence, $24000 = 27000 - 90x$.
7	Then $\phi - 90x$, $90x + 24000 = 27000$
8	Then φ 24000, $90x = 27000 - 24000$,
9	That is, $90x = 3000$,
10	That is, $9x = 300$.
11	Therefore, $x = \frac{300}{9} = 33\frac{1}{3}$,
12	And by 2*d* Step, $100 - x = 66\frac{2}{3}$.
	So that there was $33\frac{1}{3}$ Pieces, at 15 *s.* = £. 25
	And $66\frac{2}{3}$ Pieces, at 22 *s.* 6 *d.* = £. 75

PROBLEM XXVII.

A Vintner has two Vessels full of Wine, equal alike in Quantity, but of different Quality; the worst Sort is worth 240 *Crowns, and the best* 300: *Now he has another Cask or Vessel of the same Size, which he intends to fill out of these two, that when full may be worth* 260 *Crowns: How much of each must he take?*

Numerical

Numerical Solution.

1 For what he muſt take of the worſt Sort put x,

2 Then as both make but one Veſſel, the beſt will be $1 - x$.

3 Now if 1 full Veſſel be 240 Crowns, x of a Veſſel is $\frac{240x}{1}$.

4 And by the ſame Rule, $1 - x$ of a Veſſel is $\frac{300 - 300x}{1}$.

5 The Sum of the Numerators of the 3*d* and 4*th* Step is $300 - 60x$.

6 This (Q.) equal to the mean Price, 260 Crowns.

Whence this Equation, $260 = 300 - 60x$.

7 Then φ 260 and $-60x$, $60x = 300 - 260$.

8 That is, —— —— —— $60x = 40$.

9 Then muſt $6x = 4$. Therefore,

10 $x = \frac{4}{6} = \frac{2}{3}$. And by 2*d* Step,

11 $1 - x = \frac{1}{3}$.

PROOF.

So that he muſt take $\frac{2}{3}$ of the worſt, and $\frac{1}{3}$ of the beſt Sort.

For if 1 Veſſel be 240 Crowns, $\frac{2}{3}$ will be 160,
And if 1 Veſſel be 300, then $\frac{1}{3}$ will be 100,

Sum 260 = M. Price.

PRO-

PROBLEM XXVIII.

Two Men, A *and* B, *ſet out from a certain Place, the one goes* 21 *Miles in* 15 *Hours, and* 8 *Hours after he ſet out,* B *begins to travel, and goes at the Rate of* 15 *Miles in* 9 *Hours: I demand how long it will be before* B *overtakes* A, *and how far they will both have travelled?*

Numerical Solution.

1 For the Hours A travelled put x,
2 Then will B travel $x - 8$.
3 Then, If 15 Hours be 21 Miles, what will x be *Anſ.* $\frac{21x}{15}$.
4 And, If 9 Hours be 15 Miles, $x - 8$ will be $\frac{15x - 120}{9}$.
5 Now ſeeing that after B overtakes A, the Diſtance they travelled were both alike, therefore $\frac{15x - 120}{9} = \frac{21x}{15}$.
6 This reduced firſt by 9, is $15x - 120 = \frac{189x}{15}$.
7 This $\times$ 15, is $225x - 1800 = 189x$.
8 Then ϕ $189x$, $225x - 189x = 1800$,
9 That is, $36x = 1800$,
10 That is $x = \frac{1800}{36} = 50$ Hours, A.
11 And by 2*d* Step, $x - 8 = 42$ Hours, B,
12 Now A goes by 3*d* Step $\frac{21x}{15}$, that is, 70 Miles $= \frac{15x - 120}{9} = 70$, B.

PRO-

PROBLEM XXIX. *

What Number is that *that* $\frac{3}{4}$ *of it more* 12 *is equal to* $\frac{2}{3}$ *of it more* 14?

1 For the Number fought put x,

2 Then $\frac{3}{4} + 12$, is $\frac{3x}{4} + 12$,

3 And $\frac{2}{3} + 14$, is $\frac{2x}{3} + 14$.

4 Thefe (Q.) are equal.

Whence $\frac{3x}{4} + 12 = \frac{2x}{3} + 14$.

5 This reduced firft, is $3x + 48 = \frac{8x}{3} + 56$,

6 That is, being again reduced,
$9x + 144 = 8x + 168$.

7 Then φ $8x$ and $+ 144$, $9x - 8x = 168 - 144$,

8 That is, $x = 168 - 144 = 24$ *Anf.*

And this is proved at large in *Dial.* 8. *Sect.* 4. *Ex.* 5.

PROBLEM XXX.

Four Highwaymen, A, B, C, *and* D, *robb'd a Gentleman upon the Road of* 475 £. *and going to an Inn to part the Money, which they had laid upon the Table, Words arofe, and every one fnatch'd up what he could; after which, upon telling each one his Money, it was found, that if to what* A *fnatch'd up were added* 4 £. *and from* B's *were taken* 4, *and* C's *multiplied by* 4, *and* D's *divided by* 4, *it would produce one and the fame Number of Pounds: It is demanded what each fnatch'd up?*

☞ Firft,

* See the Note in the Proof of *Example* 5, in *Multiplication* of *Equations.*

☞ First, Suppose *A* snatch'd x Pounds, then having 4 added, it would be $x + 4$. Now since by subtracting 4 from *B*'s, his would be equal to *A*'s, he must then have $x + 8$; and since by multiplying *C*'s by 4, he would have the same, he must of Course snatch $\frac{x}{4} + 1$, which $\times$ 4, produces $x + 4$; and as *D*'s is to be the same if divided by 4, he must snatch $4x + 16$.

Numerical Solution.

1	For what *A* snatch'd put x,	
2	Then will *B*'s be as above —	$x + 8$,
3	*C*'s — — — — —	$\frac{x}{4} + 1$,
4	*D*'s — — — — —	$4x + 16$,
5	The Sum is ——	$6x + \frac{x}{4} + 25$.
6	This (Q) is equal to the Robbery.	
	Whence,	$6x + \frac{x}{4} + 25 = 475$.
7	This reduced,	$24x + x = 100 + 1900$.
8	Then ϕ 100,	$25x = 1900 - 100 = 1800$.
9	Therefore,	$x = \frac{1800}{25} = 72$, *A*,
10	And by 2*d* Step,	$x + 8 = 80$, *B*,
11	And by the 3*d*,	$\frac{x}{4} + 1 = 19$, *C*.
12	And by the 4*th*,	$4x + 16 = 304$, *D*,
		Sum 475.

Tyr. This is a hard Question I think; at least I thought so at first reading.

Phi.

Phi. It may be ſo; and yet in other Words it may be eaſy; for it is no other than this: What 4 Numbers muſt 475 be divided into, ſo that the 1*ſt* having 4 added to it, the 2*d* 4 taken from it, the 3*d* multiplied by 4, and the 4*th* divided by 4, may be all equal? Which Numbers are as before, *viz.* 72, 80, 19, and 304; which you may prove.

PROBLEM XXXI.

Two Graziers, A *and* B, *coming from a Fair, were met by two Highwaymen, who robbed* A *of* 25 £. 10 *s. and* B *of* 7 £. 10 *s. but upon their complaining that they had a great many Miles to ride, and Nothing to bear their Charges, he that robbed* A *return'd him a certain Sum, and ſo did the other to* B. *Now after* A *come to tell* B *what he had left, and* B *come to tell* A, *it was diſcovered that they robbed* A *of* 3 *Times as much as* B, *and left* B $\frac{1}{5}$ *of what they left* A: *I demand what each was really robbed of?*

Numerical Solution.

1	For what they took from *B* put x,	
2	Then will what they took from *A* be $3x$	
3	Now *B* had £. 7, 10 *s.* or 150 *s.* at firſt,	
	But now, —— ——	$150 - x$ left,
4	Alſo *A* had £ 25, 10 *s.* or 510 *s.* at firſt,	
	But now, —— ——	$510 - 3x$ left.
5	This (Q.) ſhould be 5 Times what *B* had left.	
	Whence,	$510 - 3x\ A = 750 - 5x\ B.$
6	Then $\phi - 5x$,	$510 + 5x - 3x = 750.$
7	Then ϕ 510,	$5x - 3x = 750 - 510,$
8	That is, —— ——	$2x = 240.$
9	Therefore,	$x = \frac{240}{2} = 120\,s.\ B.$

10	And by the 2*d* Step, $3x = 360$, *A*.
	So that they took £. 18 from *A*, and 6 from *B*.

PROBLEM XXXII.

A General of an Army had a certain Number of Men, which he intended to place in a ſquare Battalia, but diſpoſing of them in Rank and File, found he had 90 Men to ſpare; now thinking to get theſe in alſo, he enlarged his Square to one Man more in Rank and File, but then found he wanted 39 Men to complete the Square: What Number of Men had he, and how many ſtood in Rank and File?

Numerical Solution.

1	For the N°. that ſtood in Rank and File put x,
2	Then will the Square of theſe be xx,
3	But having 90 to ſpare, he had $xx + 90$.
4	Now encreaſing Rank and File by 1, the Side is — — — $x + 1$,
5	The Square of which is $xx + 2x + 1$,
6	From this take $xx + 90$, there remains $2x - 89$.
7	This (Q.) is equal to 39.
	Whence, — — $2x - 89 = 39$.
8	Then $\varphi - 89$, — $2x = 39 + 89$,
9	That is — — $2x = 128$.
10	Therefore, — $x = 64$ a Side.

PROOF.

There were 64 Men in Rank, and 64 in File, and $64 \times 64 = 4096$, to which add 90, it is 4186. But had 65 Men been the Side of the Square, there would have been 4225 Men, which is 39 more than 4186, Q. E. D.

PRO-

* PROBLEM XXXIII.

There is a Veſſel (partly) empty in which are 20 *Gallons of Wine, worth* 8 s. per *Gallon; now if it be filled up with Water, the Wine and the Water together will be worth* 6 s. *a Gallon, and the Whole worth the ſame Money as when it was all Wine: I demand then what the Veſſel holds when full, or, which is the ſame, how many Gallons of Water will fill it up?*

Numerical Solution.

1 For what the Veſſel wants in Gallons put x,

2 Now 20 Gallons of Wine, at 8 *s.* is 160 *s.*

3 Now when the Veſſel is filled up, the Whole will fetch the ſame as the Whole of the Wine did, *viz.* 160 *s.*

4 Then ſay, If $20 + x$ be 160 *s.* what is 1 Gallon? —— —— *Anſ.* $\frac{160}{20 + x}$,

5 This (Q) is equal to 6 *s.* Whence, $6 = \frac{160}{20+x}$,

6 That is, —— $120 + 6x = 160$.

7 Then φ 120, $6x = 160 - 120 = 40$.

8 Therefore —— $x = \frac{40}{6} = 6\frac{2}{3}$.

So that there wanted $6\frac{2}{3}$ of Water to fill it: To which add Wine 20 Gallons, it is $26\frac{2}{3}$ Gallons the Contents.

PROOF.

	£.	s.
20 Gallons of Wine, at 8 *s.* =	8	0
And $26\frac{2}{3}$ of mixt, at 6 *s.* =	8	0

* PROBLEM XXXIV.

Vertruvius, (Lib. ix. Chap. 3.) *informs us, That King* Hiero *being obliged by Vow to make a Preſent of a Crown of pure Gold, weighing* 100 ℔. *gave Orders for ſuch an one to be made; but being told that the Goldſmith had ſecreted Part of the Gold, and put to the Crown the ſame Weight of Silver, he ſent for the famous* Archimides *of* Syracuſe, *to whom he recommended the Diſcovery of the Fraud: It is demanded how* Archimides *diſcovered the Cheat, and how many Pounds of Silver the Goldſmith had put into the Crown?*

☞ Since it is proved by Experiment that a Maſs of pure Gold will poſſeſs leſs Space than a Quantity of Silver of the ſame Weight, it will be eaſy to conceive that a mixt Maſs of Silver will poſſeſs, or take up a Space between them. *Virtruvius* therefore cauſed two Maſſes to be made of equal Weight with the Crown, one all of pure Gold, and the other all Silver; then having a Veſſel filled to the Brim with Water, he cauſed the Crown to be immerged, carefully reſerving the Water which flowed over. And thus he did with the Maſs of Gold and Silver, reſerving each Time the Water which flowed over the Veſſel; by which Means he very exactly told *Hiero* how much Gold was ſecreted.

Now let us ſuppoſe, that by immerging the Maſs of Gold only, there was emitted 60 ℔ of Water, and by the Maſs of Silver 90 ℔, and by the Crown 64 ℔. Then,

Numerical Solution.

1	For the Quantity of Siver in the Crown put x,
2	Then muſt the Gold —— $100 - x$.

3 Now if 100 ℔. of Gold be 60 ℔. Water, $100 - x$ is $\frac{6000 + 30x}{100.}$

4 Again, If 100 ℔. of Silver be 90 ℔. Water, x is $\frac{90x}{100}$.

5 The Sum of the two is $\frac{6000 + 30x}{100.}$

6 This ought then to be equal to the Pounds of Water emitted by the Crown, *viz.* 64.

Whence, $\frac{6000 + 30x}{100} = 64$.

7 This reduced, —— $6000 + 30x = 6400$,

8 That is, —— $30x = 6400 - 6000$,

9 That is, —— $30x = 400$

10 That is, —— —— $3x = 40$

11 Therefore, —— $x = \frac{40}{3} = 13\frac{1}{3}$.

So that the Goldsmith had mixt $13\frac{1}{3}$ ℔. of Silver in the Crown.

N. B. But there may be an infinite Number of Anſwers produced, according to the Variation of the Queſtion, and the Crown will be more or leſs adulterated, according to the Proportion of the Water emitted by the Maſs of Gold and the mixt Maſs; for the leſs their Difference, the leſs the Adulteration.

Note alſo, That this and ſuch like Queſtions may be done by knowing the ſpecific Gravity of each Body; that is, weighed ſeparately in Air and Water, and their Proportions will hold good in the ſame Manner as above, and is more modernly practiſed.

PROBLEM XXXV.

What Number is that whose $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ exceeds the Whole by 240?

Numerical Solution.

1 For the Number put x,

2 Then will its $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, be $\frac{x}{2}$, $\frac{x}{3}$, $\frac{x}{4}$.

3 These reduced to a C. D. will be $\frac{12x+8x+6x}{24}$, or rather, $\frac{26x}{24}=\frac{13x}{12}$.

4 Whence (Q.) this Equation, $\frac{13x}{12}=x+240$.

5 This reduced, —— $13x=12x+2880$.

6 And φ $12x$, —— $13x-12x=2880$,

7 That is, —— $x=2880$, the Number.

PROBLEM XXXVI.

There is a Rod of Iron a Yard long, at the Ends of which hang 2 Weights, viz. *one of* 15, *and the other of* 1 ℔. *Weight: I demand the Point of the Rod where these two Weights will hang in Balance?*

Numerical Solution.

1 For the Distance of the less Weight to the Point put x,

2 Then will the Remainder be the Distance of the Greater, *viz.* $36-x$ Inches.

3 Then will the Proportion be, As $x : 36-x :: 15 : 1$.

4 Multiplying Means and Ext. $1x=540-15x$,

5 Then φ $-15x$, —— —— $16x=540$.

| 6 |

6 Therefore, —— $x = \frac{540}{16} = 33\frac{3}{4}$. Lefs Wt.

7 And by 2*d* Step, $36 - x = 2\frac{1}{4}$, Greater.
So that the Point from the fmall Weight muft be $33\frac{3}{4}$ Inches, and the Diftance of the 15 ℔. Weight 15 Times lefs, *viz.* $2\frac{1}{4}$ from the End.

PROBLEM XXXVII.

Suppofe a Rod of Iron to be equally divided into 150 *equal Parts, and at the firft Part or Divifion hangs a* 4 ℔. *Weight, and on the laft Divifion, or other End, a Weight of* 4 *Score and* 16 *Pounds: I demand the Point of the Rod where thefe two Weights will be in* Equilibrio; *or, which is the fame, what Divifion of the Rod will be a Balance to both the Weights?*

Numerical Solution.

1 For the Difference of the greater Weight put x,

2 Then will the lefs be $150 - x$.

3 Then. —— As $x : 150 - x :: 4 : 96$.

4 By multiplying Means and Ex. $96x = 600 - 4x$.

5 Then $\varphi - 4x$, —— —— $100x = 600$.

6 Therefore, $x = \frac{600}{100} = 6$, greater Weight.

7 And by 2*d* Step, $150 - x = 144$, Lefs Weight.
So that the Point of Balance is 6 Parts from the greater Weight, and 144 from the Lefs: For $144 + 6 = 150$, the Whole.

☞ From hence alfo, if a Rod be divided into any Number of Parts, and one Weight be given, and the Point given in the Rod, the other Weight is eafily found by Proportion, thus: As the Diftance of one End is to the given Weight, fo is the other Diftance from the End to the required Weight.

Thus,

Thus, in the *Problem* before us, let the Weight 6 be given, and the Division 144 on which it is placed, and the whole Rod 150, as before, to find what Weight will balance it.

PROBLEM XXXVIII.

There is a Rod divided into 150 *equal Parts, on which hangs at one End a* 4 ℔. *Weight, and the Rod being placed or laid across any Thing at the* 144*th Division, I demand what Weight at the other End will be able to balance the* 4 ℔. *Weight, to keep the Rod in* Equilibrio?

This is done by the Help of the Rule of Proportion, either direct or inverse. For only observing the Distance the Point of Balance is from the given Weight, (which here is 144, and the Remainder 6) the Proportion is, As 6, the Remainder of Divisions from the less Weight, is to the Weight itself, so is the Distance of the less Weight from the Point of Balance to the greater Weight, &c. Thus,

As 6 : 4 : : 144 : 96. Or,
As 6 : 144 : : 4 : 96, &c.

☞ By this Method may be proved, whether the *Steel-yard*, or any Beam or Pair of Scales belonging be good: For notwithstanding the vulgar Notion of Beams and Scales being true, because the Brachium hangs in Balance, it is evident, that in weighing large Quantities the Buyer may be sufficiently cheated, or the Seller may ignorantly cheat himself, and that more or less, in Proportion to the Make of the Beam.

Tyr. This is a Sort of a Paradox to me at present!

Nov.

Nov. And to me likewiſe; I wiſh, therefore, *Philomathes*, you would explain it a little to me: For if it be ſo, how am I to chooſe a good Beam, or depend upon others to know whether they be true or falſe; for it is not to be ſuppoſed every one can prove it by Figures?

Phi. I grant it; but you may ſoon ſee the Truth of this by ſeveral Experiments *. However, I will tell you thus much, that in chooſing your Beam, mind not altogether its hanging in Balance; but more particularly examine whether the Arms whereon the Scales hang be equi-diſtant from the Center; for ſhould they not, you may depend it is not an honeſt Balance: And whenever you ſuſpect any Pair of Scales, you may ſatisfy yourſelf by this vulgar Experiment only; change the Weights and Commodity to the contrary Scale, and if the Weight be as before it is right, otherwiſe falſe.

Nov. We are obliged to you for this eaſy Explanation.

Phi. I am not treating of *Mechanics* it is true; but I have ſaid Something the more upon it, becauſe it is more uſeful and neceſſary in Buſineſs than every common Queſtion; and indeed, ſuch Perſons as deal in valuable Commodities in large Quantities, ſhould be careful to examine Things of this Nature. But come, we will proceed to

PROBLEM XXXIX.

A Tradeſman began the World with a certain Sum of Money, with which he bought a Stock of Goods, but by Misfortunes in Trade, he loſt the firſt Year one Half the Value of his Stock, and 10 *Shillings over; and*

* See Dr. *Deſaguliers's Experimental Philoſophy*, Vol. I. Plate 7, 8, 9, and 10.

and also the second Year he lost Half his Stock, and 10 *Shillings over: and thus he went on for* 5 *Years, losing Half the Value of the preceding Year's Stock, and* 10 *Shillings over: Now at the End of the fifth Year he left off Trade, and his Stock was worth but* 50 £. 10 *s. I demand the Value of his Stock at first?*

Numerical Solution.

1	For the Value of his Stock at first in Pounds put x,
2	Then he lost the first Year —— $\frac{x}{2}+\frac{1}{2}$ *,
3	And had left —— $\frac{x}{2}-\frac{1}{2}$, *viz.* $\frac{x-1}{2.}$
4	Then had he lost just $\frac{1}{2}$ this, the 2*d* Year it would be —— —— $\frac{x-1}{4.}$
5	But he lost 10 Shillings more, therefore he had left —— —— $\frac{x-1}{4}-\frac{1}{2}.$
6	This reduced, first $\times$ the Denominator 4, then $\times$ 2, is —— $\frac{2x-2-4}{1,}$ or, $\frac{2x-6}{8.}$
7	This abbreviated, is the 2*d* Years Stock left $\frac{x-3}{4.}$
8	Then by losing Half + 10*s.* had left the 3*d* Year, —— —— $\frac{x-3}{8}-\frac{1}{2}.$
9	This reduced, is $\frac{2x-6-8}{16,}$ or, $\frac{2x-14}{16,}$ or, $\frac{x-7}{8.}$

* Note, *As* 10 *Shillings is Half of a* £. *the* $\frac{1}{2}$ *in the* 2d *Step represents* 10 *Shillings, and saves a great deal of Trouble.*

10 | The Half of this, less 10 Shillings is left the 4*th* Year, *viz.* —— $\frac{x-7}{16} - \frac{1}{2}$. Or,

11 | This reduced, is $\frac{2x-14-16}{32}$, or, $\frac{2x-30}{32}$, or, $\frac{x-15}{16}$.

12 | Then had he left Half this, less 10 Shillings the 5*th* Year, *viz.* $\frac{x-15}{32} - \frac{1}{2}$,

13 | This reduc'd, is $\frac{2x-30-32}{64}$, or, $\frac{2x-62}{64}$, or, $\frac{x-31}{32}$.

14 | Now this (Q) is equal £. 50, 10*s.* or, $50\frac{1}{2}$.
Whence, —— $\frac{x-31}{32} = 50\frac{1}{2}$

15 | This reduced, first $\times$ Denominator 32, $x - 31 = 1600\frac{32}{2}$.

16 | This $\times$ the Denominator 2, is $2x - 62 = 3200 + 32$.

17 | Then φ — 62, $2x = 3200 + 32 + 62$.

18 | Therefore, $x = \frac{3294}{2} =$ £. 1647, his 1*st* Stock, which you may prove at Leisure.

PROBLEM XL.

What Number is that which if added severally to 3, 19, and 51, will make them 3 Proportionals?

Numerical Solution.

1 | For the Number put x,

2 | Then by adding this to each Number, they are $x + 3$, $x + 19$, $x + 51$.

3 | Whence, by the Rule of Proportion,

As

As $x + 3 : x + 19 :: x + 19 : x + 51$.

4 But multiplying *Means* and *Extremes*, you have
$$xx + 54x + 153 = xx + 38x + 361$$

5 Then by cancelling xx on both Sides,
$54x + 153 = 38x + 361$.

6 Then ϕ $38x$ and 153, it is
$54x - 38x = 361 - 153$,

7 That is, ——— $16x = 208$.

8 Therefore, $x = \frac{208}{16} = 13$ *Ans.*

PROOF.

Numbers	3	19	and	51
Add	13	13		13
	16	32		64

For $16 \times 64 = 32 \times 32$.

Do theſe Operations appear plain to you *Novitius*?

Nov. Very plain, Sir; I think I underſtand them all very well.

Tyr. I wiſh I could ſay ſo for my Part, for I muſt own at preſent I do not.

Phi. It is not to be expected ſo young a Learner as you, *Tyrunculus*, ſhould be Maſter of theſe Things at once: If you underſtand the Work by reading it, that is ſufficient at preſent, and in going through the *Problems* once more, you will, no Doubt, underſtand them; and I think I have given you a Variety of Examples enough.

Nov. I beg you would work a Queſtion or two in plain *Trigonometry* before you conclude, if *ſimple Equations* will perform them.

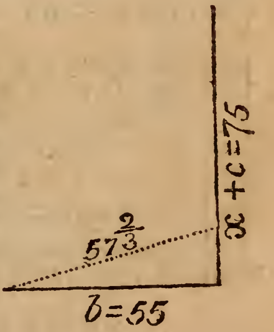
Phi. Yes, there are many to be performed by *ſimple Equations* only: But really I have ſcarce Room to

to grant your Desire, having already added ten *Problems* more than I intended. However, to oblige you I will; but then I shall work them *literally*, for it will be too tedious to do them *numerically*.

PROBLEM XLI.

Suppose a Pole to stand upon an Horizontal Plane, 75 Feet clear from the Ground (or x + c*); what Height from the Ground must it be cut or sawn off at, so that the Top of it may fall upon a Point 55 Feet from the Bottom of the Pole to the said Point on the Ground?*

Literal Solution.



1 For the Height sought put x,

2 The Square of which is xx,

3 Then is the Square of b ——— bb,

4 The Sum of their Squares $xx + bb$,

5 The Remainder to 75 is ——— $c - x$,

6 The Square of this is $cc - 2xc + xx$.

7 Whence this Equation, $xx + bb = cc - 2xc + xx$.

8 Cancel xx on both Sides, $bb = cc - 2xc$.

9 Then $\varphi - 2xc$, it is $2xc + bb = cc$.

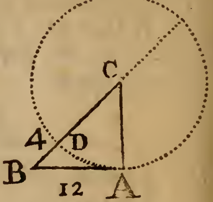
10 Then $\varphi\, bb$, ——— $2xc = cc - bb$.

11 Then must $x = \frac{cc - bb}{2c,}$ that is, $x = 75 \times 75 - 55 \times 55$, divide by 75×2, which is $= 17\frac{1}{3}$ Feet *Ans.*

PROBLEM XLII.

In the Triangle ABC *is given the Base* AB $=12$, *and the Segment, or Part of the Hypothenuse,* BC, *viz.* BD $=4$. *Required the Sides* AC *and* BC?

Literal Solution.



Let $a = 12 = AB$, and $b = 4 = BD$; and let $x = AC$. Now as AC and CD are equal, BC muſt be $= b + x$. Then by the 47*th* of *Euclid*, $BC^2 = AC^2 + AB^2$; *but* $BC^2 =$ the Square of $b + x$, *viz.* $bb + 2bx + xx$, and $aa = AB^2$

1	The Square of $b + x$ is $bb + 2bx + xx$.
2	This (Q.) is $=$ Square of x and a.
	Whence, —— $bb + 2bx + xx = xx + aa$.
3	By cancelling xx on both Sides, $bb + 2bx = aa$.
4	Then φ bb, it will be $2bx = aa - bb$.
5	Therefore, $x = \frac{aa-bb}{2b} = \frac{12 \times 12 - 4 \times 4}{2 \times 4} = 16$
	So that the Side CA or $CD = x = 16$. And CD $16 + BD = 4 = 20 =$ Hypothenuſe CB.

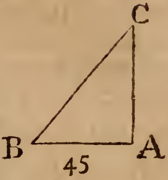
PROBLEM XLIII.

There is a rectangled Triangled ABC, *whose Base* AB $=45$, *and the Sum of the Hypothenuse and Cathetus* AC + BC $= 135$: *It is required to find the Sides* AC *and* BC *separately?*

Literal

Literal Solution.

Let $d = 135 = AC + BC$, and let $b = 45$, or BA, and put x for CA; then $CB = d - x$. And seeing the Angle CAB is a right one, we have (by the 47*th* of 1 *Euc.*) $BC^{q} = CA^{q} + BA^{q}$; but BC^{q} = the Square of $d - x$, *viz.* $dd - 2dx + xx$. And $CA = xx$; and $BA = bb$. Therefore.



1 | The Square $d - x$ is $dd - 2dx + xx$.
2 | This (Q.) = to the Square of x and b.
Whence, $dd - 2dx + xx = xx + bb$.
3 | Then cancelling xx, $dd - 2dx = bb$.
4 | Then $\oplus$ bb, ——— $dd - bb = 2dx$.
5 | Therefore, $x = \frac{dd - bb}{2d}$, or, $\frac{d}{2} - \frac{bb}{2d} = 60$ $= CA$. And $135 - 60 = 75 = BC$.

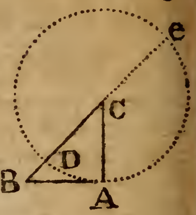
OR,

Let $x = BC$, and then will $AC = d - x$; and therefore (by the 47*th* of 1 *Euc.*) $BC^2 = AC^2 + AB^2$. Therefore, as $b = BA$, as above, it will be

1 | $xx = dd - 2dx + xx + bb$,
2 | $0 = dd - 2dx + bb$,
3 | $2dx = dd + bb$,
4 | $x = \frac{d}{2} + \frac{bb}{2d} = \frac{135}{2} + \frac{45 \times 45}{2 \times 135} = 75$ the Hypothenuse BC. And $135 - 75 = 60 = AC$, as before.

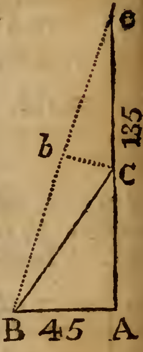
 Note,

Note, By the 36*th* of 3 *Euclid*, you may find the Sides *BC* and *AC* thus: Defcribe a Circle, making the Perpendicular the Radius; then is the Rectangle *Be* into the Segment $BD =$ the Square of *AB*; therefore *BD* is $=$ the Square of *AB* divided by $Be = BC + AC = 135$; that is, $BD = \frac{45 \times 45}{135} = 15$; and therefore (by the Figure and the preceding Work) $BC + AC$; that is, $\frac{135 - 15}{2} = 60\ AC$, and $60 + 15 = 75\ BC$.



Geometrically.

Firft, From any Scale of equal Parts make the Line $AB = 45$, and at right Angles to it draw $Ae = 135$, the Sum of the Sides *AC* and *BC*. Then join *Be* with a right Line from *e* to *B*, and divide this in the Middle at *b*; then let fall a Perpendicular from *b* upon the Line *Ae*, which will fall upon the Point *C*; then having drawn the Line *BC*, the Triangle is compleated: And if you meafure the Sides upon the fame Scale, you will find $BC = 75$, and $AC = 60$, as above: For the Line *bc* being perpendicular to *Be*, and cutting it into two equal Parts; the Triangle *BCe* is *Ifofcles*, by the 5*th* of 1 *Euc.* Therefore confequently $AC + BC = AC + Ce$. Q. E. D.



And thus, *Novitius*, I have done all that is in my Power to ferve you and *Tyrunculus*; and I fhall leave

7 *Problems* more, without their Operation, (to make up the Number 50) for your Practice; and I desire you would assist *Tyrunculus* in them, as I have assisted you in the others.

SECT. II.

Here follows some more PROBLEMS, *to exercise the young* ALGEBRAIST.

PROBLEM XLIV.

One hires a Farm containing 125 *Acres of Ground, for which he gives* 38 £. 5 *s. the Land consists of* 2 *Sorts; for the better Sort he gave* 7 s. 6 d. per *Acre, and for the worst* 3 s. 9 d. per *Acre: I demand how many Acres there were of each Sort?*

ans. 79 at 7/6
46 at 3/9.

PROBLEM XLV.

One lets out 60 £. *in* 2 *Parcels, one at* 5, *and the other at* 6 per Cent. *which in* 13 *Years simple Interest wanted but* 19 £. 7 s. 6 d *of the Principal: I demand the Parcels?*

PROBLEM XLVI.

Three Drunkards, A B, *and* C, *having each of them run up a separate Score at an Alehouse, agreed to go (under Pretence to drink) and rub all out; which was done accordingly: But the Landlord remembered very well, that* A's *and* B's *reckoning added together made* 16 s. 10 d. $\frac{1}{2}$, *and* B's *and* C's 13 s. 3 d. $\frac{1}{4}$, *and* A's *and* C's 11 s. 5 d. $\frac{3}{4}$: *He therefore craves your Assistance from hence, to tell him each Man's distinct Score?*

s d
ans. A = 7-6 $\frac{2}{4}$
B = 9-4
C = 3-11 $\frac{1}{4}$

T 3 PRO-

PROBLEM XLVII.

There are two Numbers whose Sum is 517, *and the Quotient of the Greater by the Less is just* 1000: *I demand the Numbers?*

PROBLEM XLVIII.

What Number is that, which, if added to 33, 209, *and* 561, *will make them* 3 *Proportionals?*

ans. 143.

PROBLEM XLIX.

What Number is that whose $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, *and* $\frac{1}{12}$, *exceeds itself by* 1?

ans. 3.

PROBLEM L.

A Person dying, left in Cash 410 £. 10s. *to his* 4 *Sons*, A, B, C, *and* D, *in such a Manner, that if* A *had had* 4 £. 10s. *more, and* B 4 £. 10s. *less, and* C's *were multiplied by* 4 £. 10s. *and* D's *divided by* 4 £. 10s. *it would produce one and the same Sum of Money: What was the Portion of each?*

Nov. But pray why do you not insert their Answers?

Tyr. It would be some Help I think to the young Practitioner.

Phi. It would be indulging him you might say indeed, but I cannot see why the Questions are at all the harder to be done, without it be to such who from the Answers often guess out the Numbers; therefore I choose rather to omit the Answers: For if you do the Work right, it will prove itself; and to a diligent Learner it is all the same as if the Answers were before him; and I am sure it is a properer Ex-

ercise

ercife to qualify you for more difficult Things of this Sort.

Nov. You may depend upon it, Sir, I will do my Endeavour to find their Anfwers in a fhort Time.

Tyr. So will I, as foon as I am a little more perfect in the foregoing *Problems.*

Phi. You are right, *Tyrunculus*, for from a true Knowledge of them you will foon difcover thefe alfo.

Nov. We are highly obliged to you, *Philomathes*, for thefe Favours.——Come, *Tyrunculus*, do you think of going?

Tyr. When you pleafe, Sir.

Nov. Dear *Philomathes*, in accepting my hearty Thanks you will yet more oblige your humble Servant.

Tyr. Pray receive mine alfo.

Phi. I do; and you are not only welcome to thefe fmall Inftructions, but I fhall aways be ready to ferve you: Only let me perfuade you (as far as Things of more Moment will allow of) to affift one another; for it is poffible I may (by and by) inftruct you in fomething of *Quadratic Equations*, becaufe it is a Pleafure to me to fee you delight thus in Figures. Thofe that have no Tafte for this Sort of Learning indeed, are ignorant of the Satisfaction that it leaves; for what can be a greater Satisfaction to the Mind than *Certainty* itfelf, built upon the Foundation of unerring Principles? This made a noted Author fay, that "*Algebra*, like *Logic* gives "us a juft Idea of the Nature of Things, fhews us "the true Way of reafoning, elevates the Mind to "a proper Degree, and will not fuffer it to dwell "upon mean and bafe Trifles."

And I could heartily wifh that more of the growing Youth of this Age (efpecially fuch as can afford it)

it) would (with you) give their Minds to the Study of ſome of the *Mathematical Sciences*, they being not only uſeful, but very diverting, and would certainly tend much more to their own private Good, and *that* of others, rather than the conſtant Peruſal of ſuch Books which daily vitiate the Mind, and corrupt the Morals. Thus we read, "*Xenophon* commended "the *Perſians* for their careful and prudent Educa-"tion of their Children, who made them ſtudy only "ſuch Authors as treated of Learning and Mora-"lity; but would not ſuffer them to effeminate "themſelves with idle and amorous Tales, knowing "well, and wiſely diſcerning, *that there needed no* "*Weight to be added to the* Bias *of corrupt Nature*."

N. B. Page 173, Line 23, for $\frac{29}{40}$ *read* $\frac{49}{40}$.

A N

AN

APPENDIX

CONTAINING

Some necessary INSTRUCTIONS

In the RUDIMENTS of

QUADRATIC EQUATIONS,

VIZ.

I. INVOLUTION, or the Method of raising POWERS or involving QUANTITIES.

II. The RESOLUTION of a SQUARE raised from a BENOMINAL, and how to compleat the SQUARE.

III. Of EVOLUTION or extracting ROOTS.

APPENDIX, &c.

Or a DIALOGUE between *PHILOMATHES* and *NOVITIUS*

CONTAINING

Some neceſſary INSTRUCTIONS IN QUADRATIC EQUATIONS.

PHILOMATHES *calls upon* NOVITIUS *to know what Improvement he has made in* SIMPLE EQUATIONS.

ɷ *The Sign of* INVOLUTION.

ɯ *The Sign of* EVOLUTION.

$\surd$ *The Sign of a* SURD *or* IRRATIONALITY.

Phi. TYRUNCULUS your Servant.

Nov. Sir, I am heartily glad to ſee you.

Phi. You remember I promiſed to give you ſome Notion of *Quadratics*, which I intended to have done before, but that Buſineſs of greater Moment has continually called for my Attention: And though I am

I am now come according to Promiſe; yet my Viſit will be but ſhort: And before I begin with you let me know whether you are pretty perfect in what I have ſhewn you before.

Nov. That I aſſure you I am.

Phi. We will proceed to the Point in Hand then.

Nov. Pray what does the Knowledge of *Quadratics* depend upon?

Phi. The Knowledge of *Quadratic Equations* depend upon theſe 4 Things.

1*ſt*. The Method of raiſing Powers from a ſingle Quantity.

2*dly*. The Reſolution of a Square raiſed from a *Benominal* or *Reſidual*, and how to complete the Square when two Members only are given. And

3*dly*. The Way or Method of extracting the Roots.

The firſt two of theſe are comprehended under the Name of *Involution*, or the Method of involving Quantities from any given Root.

Of raiſing Powers from a ſingle Quantity.

RULE.

Multiply the given Quantity into itſelf you have the Square, to which join the ſaid Quantity you have the Cube or third Power, &c. &c.

EXAMPLE I.

Let x the Root be involved to the 2, 3, 4 and 5*th* Power.

x Root.
xx Square or 2*d*. Power.
xxx Cube or 3*d*. Power.
$xxxx$ Biquadrate or 4*th*. Power.
$xxxxx$ Surſolid or 5*th* Power.

Nov.

Nov. This is ſo plain, more Examples are needleſs: But what do you mean by a *Binominal* and *Riſidual* Root?

Phi. A *Binominal* is a compound Quantity conſiſting of two Parts, as $x+b$ or $x+\frac{b}{4}$ connected together by the Signs more $+$ and leſs $-$ or $+$ or $-$ and alſo $x-b$ $x-\frac{b}{4}$. Now theſe two Parts multiplied by themſelves (that is ſquared) will always produce 3 Members, the firſt and laſt of which will be perfect Squares of the Root itſelf, and will always be *affirmative*; and the middle Part or Member is made by the double Rectangle of the Parts of which the *Binominal* is compoſed, and this middle Part will be ſometimes *Affirmative*, and ſometimes *Negative*: *Affirmative* when both are *Affirmative* or *Negative*, as $x+b$ or $-x-b$; and *Negative* when one of the Parts are *Negative*, as $x-b$ *; do you underſtand me?

Nov. Yes very well, except it be the double Rectangle you talk of.

Phi. This I ſhall ſatisfy you about preſently, under Obſervation the Firſt: In the mean Time, we will give you an Example.

2*dly*. The Reſolution of a Square raiſed from a *Binominal* or *Riſidual*, and how to complete the Square when two Members only are given.

* *Note*, A *Binominal* is called a *Riſidual* Root when one Part is negative as $x-b$.

EXAMPLE 2.

Let $x + b$ a *Binominal* be raiſed or involved to the third Power.

$$\begin{array}{l} x + b \\ x + b \\ \hline xx + xb \\ \quad\quad xb + bb \\ \hline xx + 2xb + bb \end{array}$$

the Square. This $\times\ x + b$

$$\begin{array}{l} \quad x + b \\ \hline xxx + 2xxb + xbb \\ \quad\quad\quad\quad xxb + 2xbb + bbb \\ \hline xxx + 3xxb + 3xbb + bbb \end{array}$$

Cube.

EXAMPLE 3.

Let $x - b$ a *Riſidual* be raiſed or involved to the third Power.

$$\begin{array}{l} x - b \\ x - b \\ \hline xx - xb \\ \quad - xb + bb \\ \hline xx - 2xb + bb \end{array}$$

Square $\times\ x - b$

$$\begin{array}{l} \quad x - b \\ xxx - 2xxb + xbb \\ \hline \quad\quad - xxb + 2xbb - bbb \\ \hline xxx - 3xxb + 3xbb - bbb \end{array}$$

Cube.

Nov.

Nov. I underſtand you very well; and I alſo perceive that in the *Binominal* the Anſwer is affirmative in all the Quantities; but in the *Riſidual* they alternately change.

Phi. You ſay right; and you ſee that it is Nothing but common Multiplication; and if there were Fractions the Work is the ſame.

EXAMPLE 4.

Thus, $\frac{x}{2}$ ſquared is $\frac{xx}{4}$, for only multiply the Denominators and the Numerators together as in Multiplication of Algebraic Fractions, ſo alſo $\frac{x}{4}$ ſquared $\frac{xx}{10}$; and $\frac{x}{4} + \frac{b}{2}$ ſquared is $\frac{xb}{8}$; and $x + \frac{a}{2}$ a *Binominal* is $xx + \frac{2ax}{2} + \frac{aa}{4}$, or $xx + ax + \frac{aa}{4}$. And laſtly, $x - \frac{a}{2}$ will alſo be $xx - \frac{2ax}{2} + \frac{aa}{4}$, or $xx - ax + \frac{aa}{4}$. For 2 in the middle Term being common to the Numerator and Denominator I expunge it, and take the Numerator ax only.

Tyr. I thank you kind Sir, for your demonſtrating it ſo plainly.

Phi. I preſume then, as you know how to involve any Root, you alſo know the third Thing, that is, how to complete the Square.

Nov. That I do not, nor do I know altogether at preſent what you mean.

Phi. I own as I ſaid before, that it is not ſo eaſily known by a Learner; therefore I readily excuſe it, becauſe Authors in general have taken *no* Notice of this, though ſo neceſſary.

Of compleating the Square when but two Members are given.

I have already told you, that any compound Quantity, whether *Binominal* or *Rifidual*, when fquared, will confift of thefe Members; the Middle of which will be fometimes *affirmative*, and fometimes *negative*: I alfo told you, that when they are both perfect Squares you may eafily know it, as $xx + bb$ or $xx + 16$; to compleat the Square of which will eafily appear as follows.

OBSERV. I.

When any compound Quantity as $xx + bb$ or $xx - bb$ wants to be compleated, that is, wants the middle Part or Member, (for remember I told you it confifts of 3) then take the Root of each Part, *viz.* x and b, and multiply them together, which is xb or $-xb$, this is what we call the Rectangle, the Double of which is $2xb$ or $-2xb$, either of thefe put between the other two will compleat the Square.

Nov. This is eafy indeed if I take it right; for fuppofe $xx + bb$ were to be compleated, the Root of xx is x the Root of bb is b; now $x \times b = xb$ the fingle Rectangle, therefore $2xb$ is the double Rectangle, which placed between the other two Members will be $xx + 2xb + bb$, or $xx - 2xb + bb$. But how am I to proceed when there are Co efficients?

Phi. The very fame: For fuppofe the Root $8x - 2$ were to be fquared or compleated, here $8x \times 8x = 64x$ the firft Term, and $-2 \times -2 = 4$ the third Term. Now $8x \times -2 = -16x$ the fingle Rectangle, therefore $-32x$ is the double Rectangle or middle Term, fo the Square compleated is $64x - 32x + 4$.

Numerical

Numerical Demonſtration.

Suppoſe they were in Numbers only, you will ſee the middle Term is always made up with the double Rectangle of the Parts. For let any Number (ſuppoſe 16) be divided into any two Parts, as 12 and 4. To compleat the Square $4 \times 12 = 48$, which doubled, is 96 the middle Term, ſo is the Square compleated, *viz.* $144 + 96 + 16$; and if you make the *Binominal* $x + b = 12 + 4$, you will in courſe have $xx + 2xb + bb = 144 + 96 + 16$.

Nov. I like this very well; but pray how do you compleat the Square? when of the two given Members only, the firſt is a Square, and the other any Quantity or Number propoſed at random.

Phi. To be ſure this cannot be done in many Caſes, by taking the double Rectangle of the Parts as before directed, becauſe the Parts conſiſt not of two pure or perfect Squares; but ſtill, *Tyrunculus*, we ſhall put you into an eaſy Way of doing it at once, I'll warrant you, by the third Thing propoſed, namely,

OBSERV. 2.

When any two Quantities are propoſed, whereof the firſt is a Square, whether they have Co-efficients or not, or whether the ſecond Member be a Fraction or not, you may find the third Member and compleat the Square of two ſuch Quantities by this Rule only.

Another general Rule to complete the Square of any two Quantities, one whereof is a perfect Square.

RULE.

Take half the Co-efficient of the ſecond Member, and the Square thereof ſhall be the third Member,

which will compleat the Square of the ſaid two given Members.

Tyr. What do you ſay this will do, though I propoſe any Quantities or Fractions at Random?

Phi. Yes, provided your firſt Member be a perfect Square, and the ſecond has the Root of that Square found in it.

Tyr. Give me a few Examples.

Phi. I will.

EXAMPLE 1.

Suppoſe $xx + 8x$ were to be compleated here, half the Co-efficient of the ſecond Member, *viz.* 8 is to this ſquared is 16 which will be the third Term. $xx + 8x$ therefore when compleated, will be $xx + 8x + 16$, the Root of which is $x + 4$, for $x + 4 \times x + 4 = xx + 8x + 16$.

EXAMPLE 2.

Let $xx + 14x$ be compleated.

Here half of 14 is 7, this ſquared is 49; ſo that $xx + 14x$ when compleated is $xx + 14x + 49$.

Nov. Very eaſy indeed, and very pretty.

Phi. Notwithſtanding this you ſhall very rarely meet with it in Authors.

Nov. I know it; but pray ſuppoſe the ſecond Member have an odd Number or Co-efficient, or ſuppoſe it to have Fractions how then?

Phi. The very ſame as before.

EXAMPLE 3.

Let $xx + 5x$ be compleated.

Nov. I am at a Loſs at preſent indeed.

Phi. Surely not, *Tyrunculus!* why is not the Half of 5 expreſſed $\frac{5}{2}$?

Nov.

Nov. I aſk Pardon, it is ſo, and the Square of $\frac{5}{2}$ is $\frac{25}{4}$, is it not?

Phi. Be ſure it is.

Nov. Then I perceive $xx + 5x$ when compleated, will be $xx + 5x + \frac{25}{4}$. And by the ſame Rule $xx - \frac{2x}{5}$ will be $xx - \frac{2x}{5} - \frac{1}{25}$, for the $\frac{1}{2}$ of $\frac{2}{5}$ is $\frac{1}{5}$ and the Square of $\frac{1}{5}$ is $\frac{1}{25}$, ſhew me one or two literally.

EXAMPLE 5.

Suppoſe $xx + bx$ be given, what will the third Member be to compleat the Square?

Here the Co-efficients of the ſecond Member is b, Half of which is $\frac{b}{2}$, which ſquared, is $\frac{bb}{4}$, ſo that $xx + bx$ when compleated $xx + bx + \frac{bb}{4}$. See *Page* 219, *Example* 4.

EXAMPLE 6.

$xx + \frac{bx}{a}$ when compleated, is $xx + \frac{bx}{a} + \frac{bb}{4aa}$.

For half the Co-efficient $\frac{b}{a}$ is $\frac{b}{2a}$ the Square of which is $\frac{bb}{4aa}$.

Are you ſenſible of this?

Nov. Nothing appears plainer.

Phi. Since you know Something of the Nature of *Involution*, and compleating the Square, I will now give you a Notion of *Evolution* directly.

3. *Of* EVOLUTION.

Evolution is the Reverſe of *Involution*, and ſhews us how to extract the Roots of any given Power.

Ex-

EXAMPLES.

$$\begin{array}{ll} xx & xxxx \\ \quad x \text{ Root} & \quad xx \text{ Root.} \\ xx\ bb\ cc & xx\ bbbb.\ dddd \\ x\ \ b\ \ c \text{ Root} & x\ \ bb\ \ \ dd \text{ Root.} \end{array}$$

OBSERV. I.

When there are ſeveral Quantities in one Power, then conſider which of thoſe Powers are perfect or pure Squares of themſelves; for ſhould the firſt and third be ſo in any Power raiſed from a *Binominal* or *Riſidual*, extract the Root of the ſaid two Powers, and you have the ſquare Root of the whole Quantity or Power. Thus,

$$\frac{xx + 2xb + 2bb \text{ Square.}}{x + b \qquad \text{Root.}}$$

For the Root of xx is x, and the Root of bb is b, and theſe two connected are $x + b$, and theſe I ſuppoſe the true Root; but I find it to be ſo upon two Trials, firſt $x \times b = ab$, this doubled is $2xb$ the middle Term; alſo $x + b \times x + b$ gives $xx + 2xb + bb$ the Power given. Again,

$$x^4 + 6\ xxx + 9\ xx \quad \text{Square.}$$
$$xx + 3\ x \quad \text{Root.}$$

Alſo ſuppoſe $xxxx - 14\ xxbb\ cc + 49\ bbbb.cccc.$
Then $xx - 7\ bb\ cc$ Root.

Here are two pure Powers, the Square of which is xx and $7\ bb\ cc$; therefore I conclude $xx - bb\ cc$ the Root, becauſe the middle Member is *negative*, and the

the Square of half its Co-efficient gives 49 in the third Member.

Nov. I underſtand you well; but how am I to extract the ſquare Root of Fractions?

Phi. After the ſame Manner. For,

OBSERV. 2.

If the firſt Member be a pure Power, and the Fraction alſo, you may conſider it as a perfect Square raiſed from a *Binominal* or *Riſidual* Root; extract therefore the Root of the Numerator for a new Numerator, and of the Denominator for a new Denominator.

EXAMPLES.

Let $xx + 3x + \frac{9}{4}$ Square
$x + \frac{3}{2}$ Root.

For the Root of $\frac{9}{4}$ is $\frac{3}{2}$; and $x + \frac{3}{2} \times x + \frac{3}{2} = xx + 3x + \frac{9}{4}$. Again,

Let $xx + 3bx + \frac{9}{4}bb$ Square
$x + \frac{3}{2}b$ Root. Again,

Let $xx - \frac{bx}{a} + \frac{bb}{4aa}$ Square
$x + \frac{b}{2a}$ Root.

See *Example* 6. in *Involution.* Alſo,

Let $xx - dx + \frac{1}{4}dd$ Square
$x - \frac{1}{2}d$ Root.

Do you underſtand it?

Tyr,

Tyr. Yes very well, except in one Thing, and that ſeems very odd to me.

Phi. What is that?

Nov. Why, I perceive the Root of the Fraction is larger than the Fraction itſelf.

Phi. Not in every Reſpect neither; for $\frac{1}{4}$ of *dd* muſt be more then $\frac{1}{2}$ *d*; but I ſuppoſe you wonder that the Square Root of $\frac{1}{4}$ ſhould be $\frac{1}{2}$, which is more than $\frac{1}{4}$ itſelf.

Nov. I do ſo.

Phi. That the Root of every ſimple Fraction is greater than the Square itſelf; you may ſee the Reaſon of this, in *Dialogue* 3. *Sect.* 3. *Note* 1. *and Note the* 3*d. Sect.* 4. *of the ſame Dialogue.*

Nov. But I wiſh you would demonſtrate it.

Phi. You aſk Things indeed foreign to the Purpoſe; however, I am ready to oblige you in every Thing that may be ſerviceable; I ſhall therefore explain it by Decimal Fractions; and you will ſee it at once: Now I ſuppoſe you know in Decimals .25 is $\frac{1}{4}$ .5 is $\frac{1}{2}$ and .75 is $\frac{3}{4}$ of any Thing.

Nov. Yes very well, for 25 is $\frac{1}{4}$ of 100, 5 is $\frac{1}{2}$ of 10, and 75 $\frac{3}{4}$ of 100, their reſpective Denominators.

Phi. Right, obſerve then, I only ſet down 25 as it ſtands in whole Numbers, and find the Root thereof 5, which is five Times leſs in the Root than the Square. I alſo in $\frac{1}{4}$ ſet down 25 with a Dot or Prick before it thus .25, and the Square Root is ſtill 5; but I put a Prick alſo before the .5 *that* being a Decimal alſo: Now .5 you know is $\frac{1}{2}$: By this you ſee that the Square Root of ſimple Fractions encreaſe in Value or Quantity in proportion to the Decreaſe of Roots of whole Numbers.

Nov. I ſee it plainly, and I heartily thank you; but how ſhall I know when a Square is not perfect, and how am I to act in ſuch a Caſe. *Phi.*

A TABLE *of* CONVERGING SERIES, *&c.*

Shewing by Inſpection only,

The ROOTS and POWERS of ſuch ROOTS to the Twelfth ROOT; by which any higher POWERS may be found.

Root, or Firſt Power - - - - - - - -	$x = 1$	$x = 2$	$x = 3$	$x = 4$	$x = 5$	$x = 6$	$x = 7$	$x = 8$	$x = 9$
Square, or Second Power - - - - - -	$x^2 = 1$	4	9	16	25	36	49	64	81
Cube, or Third Power - - - - - - -	$x^3 = 1$	8	27	64	125	216	343	512	729
Biquadrate, or Fourth Power - - - -	$x^4 = 1$	16	81	256	625	1296	2401	4096	6561
Surſolid, or Fifth Power - - - - - -	$x^5 = 1$	32	243	1024	3125	7776	16807	32768	59049
Square-cubed, or Sixth Power - - - -	$^6 = 1$	64	729	4096	15625	46656	117649	262144	531441
Second Surſolids, or Seventh Power - -	$x^7 = 1$	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrate ſquared, or Eighth Power	$x^8 = 1$	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cube cubed, or Ninth Power - - - -	$x^9 = 1$	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Surſolid ſquared, or Tenth Power - -	$x^{10} = 1$	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
Third Surſolid, or Eleventh Power -	$x^{11} = 1$	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
Square-Cube ſquared, or Twelfth Power	$x^{12} = 1$	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481

Let this fold againſt Page 227.

Phi. That will diſcover itſelf by the foregoing Rules; that is, if there be not two pure Squares; or if the double Rectangle under the Squares make not the middle Term: In ſuch Caſes as theſe, you only put this Sign ($\surd$) before it, to ſhew it is a ſurd Quantity. For this Sign is called the *Radical Sign*, or the Sign of *Irrationality*. Thus,

The ſquare Root of xb is $\sqrt{xb}$ of $xx + dd$ is $\sqrt{xx + dd}$.

Now you ſee it is plain $xx + dd$ is abſurd, or a Quantity that is not a perfect Square; for the Square of xx is x, and the Square of dd is d; theſe connected are $x + d$; but $x + d \times x + d = xx + 2xd + dd$ conſequently therefore $xx + dd$ is a ſurd Quantity.

So alſo the ſquare Root of $xx + 2xb - bb$ is $\sqrt{xx + 2xb - bb}$, becauſe the firſt is *affirmative*, and the third *negative*.

Again, The ſquare Root of $xx + 5xb + bb$ is expreſſed $\sqrt{xx + 5xb + bb}$; becauſe the middle Quantity is not juſt the Double of the Products of x and b.

And now, *Novitius*, I will give you a Table of the Powers, and ſhew you the Manner of involving them more plainly; and alſo more of the Nature of Inveſtigation by Way of Exerciſe.

Here follows the Method of Inveſtigation or extracting the Roots of all Powers.

THE Square Root having been ſpoken of before, I ſhall here begin with the third Power or Cube Root. And you are to take Notice, *Novitius*, *that in*

in all the following Operations wherein e *is above the ſecond Power,* that *Part muſt be rejected.*

1. *Of the* CUBE ROOT.

Let the given Number whoſe Root is to be extracted, be $= b$, and let $x + e = \sqrt[3]{b}$: Then if you involve $x + e$ to the third Power, that is, Cube it, you will have

	1	$x^3 + 3x^2e + 3xe^2 + xxx = b$
$1 \div 3x$	2	$\frac{x^2}{3} + xe + e^2 = \frac{b}{3x}$
$2 - \frac{ax}{12}$	3	$\frac{x^2}{4} + xe + e^2 = \frac{b}{3x} - \frac{xx}{12}$
3 w 2	4	$\frac{x}{2} + e = \sqrt{\frac{b}{3x} - \frac{xx}{12}}$
$4 + \frac{x}{2}$	5	$x + e = \frac{x}{2} + \sqrt{\frac{b}{3x} - \frac{xx}{12}} = \sqrt[3]{b}.$

Now this Method will always hold good in every Operation, whether you ſuppoſe $x +$ or $-$ than it really is, as plainly appears from the next Work. For,

Let $x - e = \sqrt[3]{b}$

	1	$x^3 - 3x^2e + 3xe^2 - e^3 = b$
$1 \div 3x$	2	$\frac{x^2}{3} - xe + e^2 = \frac{b}{3x}$
$2 - \frac{xx}{12}$	3	$\frac{x^2}{4} - xe + e^2 = \frac{b}{3x} - \frac{xx}{12}$
3 w 2	4	$\frac{x}{2} - e = \sqrt{\frac{b}{3x} - \frac{xx}{12}}$
$4 + \frac{x}{2}$	5	$x - e = \frac{x}{2} + \sqrt{\frac{b}{3x} - \frac{xx}{12}} = \sqrt[3]{b},$

as before.

2 Of

2. *Of the* Biquadrate *or* 4*th* Power.

Let	1	$x + e = \sqrt[4]{b}$
$1 \odot 4$	2	$x^4 + 4x^3 e + 6x^2 e^2 = b$
$2 \div 6x^2$	3	$\frac{x^2}{6} + \frac{4xe}{6} + x^2 = \frac{b}{6x}$
$3 - \frac{xx}{18}$	4	$\frac{xx}{9} + \frac{4xe}{6} + ee = \frac{b}{6x^2} - \frac{xx}{18}$
$4 \omega 2$	5	$\frac{x}{3} + e = \sqrt{\frac{b}{6x} - \frac{xx}{18}}$
$5 + \frac{2x}{3}$	6	$x + e = \frac{2x}{3} + \sqrt{\frac{b}{6x^2} - \frac{xx}{18}} = \sqrt[4]{b}$

3 *Of the* Sursolid *or* 5*th* Power.

Let	1	$x + e = \sqrt[5]{b}$
$1 \odot 5$	2	$x^5 + 5x^4 e + 10x^3 e^2 = b$
$2 \div 10x^3$	3	$\frac{xx}{10} + \frac{xe}{2} + ee = \frac{b}{10x^3}$
$3 - \frac{3}{80} xx$	4	$\frac{xx}{16} + \frac{xe}{2} + ee = \frac{b}{10x^3} - \frac{3}{80} xx$
$4 \omega 2$	5	$\frac{x}{4} + e = \sqrt{\frac{b}{10x^3} - \frac{3xx}{80}}$
$5 + \frac{3x}{4}$	6	$x + e = \frac{3x}{4} + \sqrt{\frac{b}{10x^3} - \frac{3xx}{80}} = \sqrt[5]{b}.$

4 *Of the* CUBE *squared or 6th* POWER.

Let	1	$x+e=\sqrt[6]{b}$
1 ⊛ 6	2	$x^6+6x^5e+15x^4e^2=b$
$2 \div 15x^4$	3	$\frac{xx}{15}+\frac{2xe}{5}+ee=\frac{b}{15x}$
$3-\frac{2}{75}xx$	4	$\frac{xx}{25}+\frac{2xe}{5}+ee=\frac{b}{15x^4}-\frac{2xx}{75}$
4 ω 2	5	$\frac{x}{5}+e=\sqrt{\frac{b}{15x^4}-\frac{2xx}{75}}$
$5+\frac{4x}{5}$	6	$x+e=\frac{4x}{5}\sqrt{\frac{b}{15x^4}-\frac{2xx}{75}}=\sqrt[6]{b}$, &c.

And thus may you proceed to the *7th*, *8th*, *9th*, *10th*, *&c.* Powers.

Now from a due Consideration, *Novitius*, of the above Work (for I cannot expect you to be perfect in it yet) you may by comparing the Roots thus investigated form Methods for finding of general Theorems to extract the Root of any higher Power without any troublesome Operation.

Nov. I shall like to know that.

Phi. Observe then, first let us compare the four last Operations, and you will find the Fractions before *x* increase uniformly, and that the Numerator and Denominator of each is always Unity more added to each. Thus,

$\frac{1}{2}x$ $\frac{2}{3}x$ $\frac{3}{4}x$ $\frac{4}{5}x$ $\frac{5}{6}x$, *&c.* *&c.* as follows,

$$\frac{1}{2}x+\sqrt{\frac{b}{3x}-\frac{1}{12}xx}=\sqrt[3]{b}$$

$$\frac{2}{3}x+\sqrt{\frac{b}{6xx}-\frac{1}{18}xx}=\sqrt[4]{b}$$

$$\frac{3}{4}x+\sqrt{\frac{b}{10x^3}-\frac{3}{80}xx}=\sqrt[5]{b},\ \&c.$$

So

So that they follow you see in a Series $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, &c. from whence follows

$$\frac{1}{2}x + \sqrt{\frac{b}{\ } - xx}$$
$$\frac{2}{3}x + \sqrt{\ b \quad - xx}$$
$$\frac{3}{4}x + \sqrt{\frac{b}{\ } - xx}$$
$$\frac{4}{5}x + \sqrt{\frac{b}{\ } - xx}$$
$$\frac{5}{6}x + \sqrt{\frac{b}{\ } - xx}, \text{ \&c. \&c.}$$

OBSERV. 2.

Again, If you compare the Power by which b is divided, you will find it increased by the continual Multiplication of x; and that the Co-efficient of the said Power (b) is increased also by the continual Addition of 3, 4, 5, 6, 7, 8, &c. Hence therefore evidently arises

$$\frac{1}{2}x + \sqrt{\frac{b}{3x} - xx}$$
$$\frac{2}{3}x + \sqrt{\frac{b}{6x^2} - xx}$$
$$\frac{3}{4}x + \sqrt{\frac{b}{10x^3} - xx}$$
$$\frac{4}{5}x + \sqrt{\frac{b}{15x^4} - xx}$$
$$\frac{5}{6}x + \sqrt{\frac{b}{21x^5} - xx}, \text{ \&c. \&c.}$$

OBSERV. 3.

From what has been obſerved, it may eaſily be conceived, that the Fraction, into which xx is multiplied, are found and produced by multiplying half the Fraction annexed to x into the whole Fraction annexed to b. Whence follows,

$$\frac{x}{2} + \sqrt{\frac{b}{3x} - \frac{1}{12}xx}$$

$$\frac{2x}{3} + \sqrt{\frac{b}{6x^2} - \frac{1}{18}xx}$$

$$\frac{3x}{4} + \sqrt{\frac{b}{10x^3} - \frac{3}{80}xx}$$

$$\frac{4x}{5} + \sqrt{\frac{b}{15x^4} - \frac{2}{75}xx}$$

$$\frac{5x}{6} + \sqrt{\frac{b}{21x^5} - \frac{5}{252}xx}$$

$$\frac{6x}{7} + \sqrt{\frac{b}{28x^6} - \frac{3}{196}xx}$$

Now in order to diſcover a Theorem, by which the Root of any Power may be extracted, you are to obſerve as follows,

NOTE 1.

That the *Denominator* of the Fraction into which x is multiplied, is always leſs by Unity (or 1) than the Index of the given Power, and alſo that the *Numerator* of the ſaid Fraction is leſs by 1 than the *Denominator*.

NOTE 2.

That the Index of the Power of x, by which b is divided, is always equal to the *Numerator* of the aforeſaid Fraction.

NOTE

NOTE 3.

That the Co-efficient of the ſaid Power of x, is produced by multiplying $\frac{1}{2}$ the Index of the given Power into the ſaid Index leſs by 1.

LASTLY.

The Fraction into which xx is multiplied, is the Product of $\frac{1}{2}$ the Fraction annexed to x, and the whole numeral Fraction annexed to b.

EXAMPLE.

Let the Theorem of the $\sqrt[30]{b}$ be required, then by Note 1. $\frac{29 - 1 = 28}{30 - 1 = 29}$ Fraction of x.

And by Note 2 follows

$$\frac{b}{x28}$$

By the third Note $15 \times \overline{30 - 1} = 435$, therefore $\frac{b}{435x^{28}}$. And

By the laſt $\frac{14}{29} \times \frac{1}{435} = \frac{14}{12615}$.

The Theorem then when compleated is $\frac{28}{29}x + \sqrt{\frac{b}{435a^{28}} - \frac{14}{12615}xx} = \sqrt[30]{b}$. Again,

Let the Theorem of $\sqrt[12]{b}$ be required.

Firſt, $\frac{11 - 1 = 10}{12 - 1 = 11}x$.

2dly. $\frac{6}{10x}$.

3dly. $6 \times \overline{12 - 1} = 66$, which compleated

$$\frac{10}{11}x + \sqrt{\frac{b}{66x^{10}} - \frac{5}{726}xx} = \sqrt[12]{b}.$$

From a little Obſervation and Practice, *Novitius*, you may from theſe Examples improve yourſelf further in theſe Things. So I bid you farewel.

Nov. I beg, Sir, before you go, you would let me aſk you a particular Queſtion in Menſuration and I will trouble you no longer.

Phi. What is that pray?

Nov. Only give me your Opinion concerning the ſolid Content of different Pieces of Stone or Timber, whoſe Circumference and Length are the ſame. That is, ſuppoſe a Cylinder, and a regular Parallelopiped (having a Square for its Baſe) to be both 48 Inches Circumference and 20 Feet long, what is the Difference of their ſolid Content, or is there none?

Phi. Yes there is ſome, and a great deal too; though it is a difficult Matter to make ſome Perſons ſenſible of it who pretend to underſtand Figures well.

Nov. Pray what is the Difference?

Phi. Very near $5\frac{1}{2}$ Feet *Novitius*, the Content of the regular Parallelopiped being juſt 20 Feet, and the Cylinder 25 Feet $\frac{4}{10}$. But pray, *Novitius*, let me know the Reaſon of your aſking this Queſtion, for you ſeem to be very earneſt about it?

Nov. To tell you the Truth then, Sir, there was ſome ſmall Diſpute between two or three of us concerning it; but I could not make them ſenſible there is any Difference at all.

Phi. But why did you not work them both by Figures, and that would have convinced them?

Nov. I did, and made it the ſame as you do; but they would not be ſatisfied with *that*, which occaſioned a ſmall Bet between us to be left to your Determination.

Phi.

Phi. If you did it the right Way ſurely they could not be ſo ignorant! Let me ſee the Method of your doing it?

Nov. Firſt for the ſquare Tree, that being 48 Inches Circumference, conſequently has 12 Inches upon every Side. Now 12 multiplied by 12 makes 144, the ſuperficial Content; and this multiplied by 2c the Length, 8 divided by 144 gives juſt 20 Feet the Content.

Phi. Very right, and how did you proceed with the Cylinder?

Nov. A Cylinder having a Circle for its Baſe (and being 48 Inches Circumference) I find firſt the Diameter thus, As 3. 1416 is to 1, ſo is the Circumference (48) to the Diameter (15. 2788 Inches). Then to find the ſuperficial Content at the End, I multiply Half the Diameter (*viz.* 7. 6394) by Half the Circumference (*viz.* 24) and it gives 183. 3456 Inches the Area at the End. This divided by 144 gives 1 .2732 Feet; and this multiplied by 20 the Length, gives 25.464 ſolid Feet, which is nearly 25 $\frac{1}{2}$ as you obſerved before.

Phi. Very rightly performed; and would not this ſatisfy them do you ſay?

Nov. No indeed; they ſay all the calculated Tables in Timber-Meaſure prove the contrary: So as I obſerved before it is left to you to decide.

Phi. To oblige you, *Novitius*, I will ſhew you (by and by) a Method that will not fail to convince them. But firſt I will tell you the Reaſon of this common Error; for you muſt Note, you are not the only Perſon that have been Witneſs to this Folly. As to all the ſet Tables they are calculated for ſquare ſided Timber only, according to Cuſtom (for we are not to ſuppoſe every pretended Meaſurer a *Geometrician)* and in this the Pen and the Tables will agree

agree; and the Reaſon is this, they girt the Tree round, then take the fourth Part of that Circumference (vulgarly call'd the Girt by ſome) and multiply it by itſelf, then by the Length of the Tree; after which they divide it by 144, and it gives the Content in ſolid Feet; but then as I ſaid before, it is only for ſquare-ſided Timber that this Method holds good. For of all other ſhaped Timber the Content will be more or leſs as I ſhall demonſtrate hereafter, that will not fail, I believe, to convince your Friends of their Error.

If indeed the Buyer and Seller agree according to a cuſtomary way of meaſuring any Thing, we have no Buſineſs to meddle; but when we are called upon to do Juſtice between both; we muſt then proceed according to the juſt Rules of Arithmetic; which ought not in any Reſpect to give way to Things introduced meerly by prejudiced Ignorance, which may very well be called the Nurſes of idle Cuſtom, as you may ſee in a Series of Inſtances beſides the Caſe before us. But to give one only,

I have heard a great many pretended Meaſurers affirm; that take a round Piece of Timber and let four Slabs be ſawn off it, and even *then* it will contain more ſolid Feet than it did before. The *Engliſh* of which is, if I give you Two-pence out of a Shilling, I ſhall then have 14*d.* in Hand.——What Stupidity is here! Again, in a ſquare Tree 48 Inches round, it is plain one Side is but 12 Inches; but in a round Tree that is 48 Inches Circumference, the Side of a Square equal thereto will be 13 $\frac{1}{2}$ Inches *, but the inſcribed Square will be on each Side but about 10 $\frac{8}{10}$ Inches.

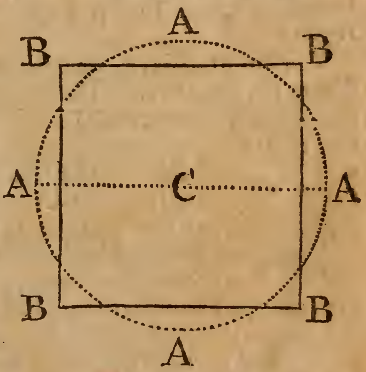
However, I ſhall leave Arithmetic and demonſtrate

* *Find the Area of the Circle, and extract the ſquare Root thereof gives the Side of the Square equal to the Circle.*

ſtrate it by one plain geometrical Figure only, which I never knew fail to convince this ſort of People; becauſe they can ſee the Reaſon of it directly upon looking at the Scheme.

Demonſtration.

As it has been proved that a Circle 48 Inches Circumference, is 15.2788 Diameter the Semi-diameter muſt be 7.6394. From any Scale of equal Parts therefore take off with your Compaſſes 7.6 Inches, and from *C* the Center, deſcribe the Circle *A*, *A*, *A*, *A*, whoſe Circumference will then be 48. Then from the ſame Scale take off 12 and make this the Side of a Square, then complete the Square *B*, *B*, *B*, *B*, whoſe Perimeter will alſo be 48 Inches. Now ſuppoſing this Square to be laid upon the Circle, does it not evidently appear by the Figure itſelf, that the *Area* or ſuperficial Content of the Circle is



larger than the Square: For though the Square hangs over the Circle at the Points, *B*, *B*, *B*, *B*, yet the 4 Areas or Segments of the Circle *A*, *A*, *A*, *A*, are each of them larger than the former. Conſequently therefore the *Area* of a Circle is larger than the *Area* of a Square, whoſe Perimeter is equal to the Circumference of the Circle; and if the ſuperficial Content be greater, it is out of Diſpute that the ſolid Content is alſo greater.

Nov.

Nov. This is a plain Demonſtration indeed !

Phi. To be ſure it is much the eaſieſt Way: For ſuch as are ignorant of the Square and Cube Root only think you are impoſing upon them when you work ſuch Queſtions at large; but here they are convinced directly.

Nov. They are ſo.

Phi. From hence then it is evident, That a Circle is larger in Area than any other Figure having the ſame Circumference. And all Polygons are nearer the Area of the Circle according to the Number of Sides; (as a Triangle, Square Pentagon, Hexagon, *&c. &c. &c.*) for the more the Sides the nearer the Circle, but they never can be quite ſo for this Reaſon, becauſe a Curve Line is longer than a ſtraight one. Again,

You are to obſerve, that the Side of the inſcribed Square (in the aforeſaid Circle) will be 10. 8 Inches, and its Content 16. 21224 Feet; and the Content of the circumſcribed Square, will be juſt the Double, *viz.* 32 .42448 Feet. The Content of the Triangle (48 Inches round) 15. 4 Feet. That of the Square juſt 20; that of the Hexagon 23 Feet.

Nov. Dear *Philomathes* I heartily thank you.

Phi. You are heartily welcome, only do you communicate to *Tyrunculus* what I have ſhewn you; for if I ſee you both diligent I intend (as ſoon as I have done with young *Tyro* in common *Arithmetic*) to inſtruct you in the Rudiments of *Geography* and the Uſe of the *Globes*. In the mean time, *Novitius*, I bid you a hearty farewel.

Nov. Sir, I am your obliged humble Servant.

FINIS.

53
48
101